CollegeBoard
Advanced Placement
Program

2008
AP® CALCULUS AB
AND
AP CALCULUS BC
RELEASED EXAMS

- Multiple-Choice Questions, Answer Keys, and Diagnostic Guides
- Free-Response Questions with Scoring Commentary, Scoring Guidelines, and Sample Student Responses
- Statistical Information About Student Performance on the 2008 Exams

Chapter I: The AP® Process

This chapter will give you a brief overview of the development and scoring processes for the AP Calculus AB and Calculus BC Exams. You can find more detailed information at AP Central® (apcentral.collegeboard.com).

What Is the Purpose of the AP Calculus AB and Calculus BC Exams?

The AP Calculus AB and Calculus BC Exams are designed to assess how well a student has mastered the concepts and techniques of calculus. The Calculus AB Exam covers topics typically included in about two-thirds of a yearlong college-level calculus sequence; the Calculus BC Exam covers topics included in a full-year, college-level calculus sequence. The multiple-choice sections of the exams are designed to test proficiency in a wide variety of topics. The free-response sections require students to demonstrate the ability to solve problems involving a more extended chain of reasoning. Both Calculus AB and Calculus BC require a similar depth of understanding of common topics, and graphing calculator use is an integral part of the courses. Both the multiple-choice and free-response sections of the exams contain parts where a graphing calculator is required and parts where calculator use is prohibited. Qualifying grades on the AP Calculus AB Exam may allow students to begin their college careers with credit for a Calculus 1 course. Qualifying grades on the AP Calculus BC Exam may allow students to begin their college careers with credit for a full year of calculus (Calculus 1 and Calculus 2). Students in both cases may also have the opportunity to register for courses for which calculus is a prerequisite.

Who Develops the Exams?

The AP Calculus Development Committee, working with mathematics Assessment Specialists at ETS, develops the exams. The committee is appointed by the College Board and is composed of seven teachers from secondary schools, colleges, and universities in the United States. The members provide different perspectives: high school teachers offer valuable advice regarding realistic expectations on matters of content coverage, skills required, levels of sophistication, and clarity of phrasing. College and university faculty ensure that the questions are at the appropriate level of difficulty for students planning to continue their studies at colleges and universities. Both high school teachers and college faculty bring technology expertise to the committee. Committee members typically serve for four years.

The Chief Reader, a college mathematics professor responsible for supervising the scoring of the free-response questions, also aids in the development process. The Chief Reader attends every committee meeting to ensure that the free-response questions selected for the exam can be scored reliably. The expertise of the Chief Reader and the committee members who have scored exams in past years is notable: they bring to bear their valuable experience from past AP Readings and suggest changes to improve the quality and the performance of the questions. In addition, the ETS Assessment Specialists offer their advice and guidance.

How Are the Exams Developed?

The Development Committee members set the specifications for the exams, determining what will be tested and how it will be tested. They also determine the appropriate level of difficulty for the exams, based on their understanding of the level of competence required for introductory calculus courses at colleges and universities. Each AP Calculus AB and Calculus BC Exam is the result of several stages of development that together span two or more years.

Section I—Multiple Choice

1. Development Committee members and external item writers compose and submit multiple-choice questions directed to the topic outlines for Calculus AB and Calculus BC in the *AP Calculus Course Description*.

2. ETS Assessment Specialists perform preliminary reviews to ensure that the multiple-choice questions are worded clearly and concisely.
3. At the committee meetings, which are held three times a year, committee members review, revise, and approve the draft questions for use on future exams. They determine whether or not a question is appropriate for testing a student's understanding of the concepts, methods, and applications of calculus. They make sure that the questions are clear and unambiguous, that each question has only one correct answer, and that the difficulty level of the questions is appropriate. For questions on the calculator-required portions of the exams, care is taken to ensure fairness regardless of the graphing calculator used.
4. From the pool of approved questions, ETS Assessment Specialists assemble draft exams according to the specifications set by the Development Committee. Many of the multiple-choice questions are pretested in high school or college calculus classes to gather data regarding the difficulty of the questions.
5. The committee members thoroughly review the draft exams in various stages of development, revising the individual questions and the mix of questions until they are satisfied with the result.

The committee helps to control the level of difficulty of the multiple-choice sections by selecting a wide range of questions, subsets of which have been used in earlier forms of the exams.

Section II—Free Response

1. Well in advance of the administration of the exams, members of the Development Committee and external item writers compose free-response questions for the exams based on the exam specifications. Appropriate combinations of questions are assembled into draft free-response sections for the exams at a committee meeting.
2. The committee members review and revise these questions at all stages of the development of the exams to ensure that they are of the highest possible quality. The committee members consider, for example, whether the questions will offer an appropriate level of difficulty and whether they will elicit answers that allow Readers, the high school and college mathematics teachers who score the free-response questions, to discriminate among the responses along an analytic scoring scale of 0 to 9 points. An ideal question enables the stronger students to demonstrate their accomplishments while revealing the limitations of less-proficient students. Each free-response question, however, is designed to have a part that is accessible for the student who is prepared to take an AP Calculus Exam.

Question Types

The 2008 AP Calculus AB Exam and Calculus BC Exam each contain a 105-minute multiple-choice section consisting of 45 questions—Part A: 55 minutes, 28 questions, no calculator allowed; Part B: 50 minutes, 17 questions, graphing calculator required. The 90-minute free-response section consists of 6 questions—Part A: 45 minutes, 3 questions, graphing calculator required; Part B: 45 minutes, 3 questions, no calculator allowed. During the second timed portion for Part B, students are permitted to continue work on the questions in Part A without the use of a calculator. The two sections are designed to complement each other and to measure a wide range of calculus concepts and skills.

Multiple-choice questions are useful for measuring a student's level of competence in a variety of contexts. In addition, they have three other strengths:

1. They are highly reliable. Reliability, or the likelihood that students of similar ability levels taking a different form of the exam will receive the same score, is controlled more effectively with multiple-choice questions than with free-response questions.
2. They allow the Development Committee to include a selection of questions at various levels of difficulty, thereby ensuring that the measurement of differences in students' achievement is optimized. For AP Exams, the most important distinctions are between students earning grades of 2 and 3 and those earning grades of 3 and 4. These distinctions are usually best accomplished by using many questions of middle difficulty.
3. They allow comparison of the ability level of the current students with those from other years. A number of questions from previous exams are included in the current one, thereby allowing comparisons to be made between the scores of the earlier group of students and those of the current group. This information, along with other data, is used to establish AP grades that reflect the competence demanded by the Advanced Placement Program® and that can be legitimately compared with grades from earlier years.

Free-response questions on the AP Calculus AB and Calculus BC Exams require students to solve problems involving a more extended chain of reasoning. The format allows students to use their analytical, reasoning, and writing skills to solve problems and present cogent answers and explanations in writing their responses. Students may be required to explain a particular concept or their methods for solving a problem, interpret a problem in context, justify their answers mathematically, or examine the reasonableness of their solutions. The two-part format for the free-response sections provides greater flexibility in the types of questions that can be given while ensuring fairness to all students taking the exams, regardless of the graphing calculator used. The free-response format allows for the presentation of uncommon yet correct responses and permits students to demonstrate their mastery of calculus by a show of creativity.

Free-response and multiple-choice questions are analyzed both individually and collectively after each administration, and the conclusions are used to improve the following year's exams.

Scoring the Exams

Who Scores the AP Calculus AB and Calculus BC Exams?

The multiple-choice answer sheets are machine scored. The teachers who score the free-response sections of the AP Calculus Exams are known as "Readers." The majority of these Readers are experienced faculty members who teach calculus at a college, university, or high school. Great care is taken to obtain a broad and balanced group of Readers. Among the factors considered before appointing someone to the role are school locale and setting (urban, rural, and so on), gender, ethnicity, and years of teaching experience. University and high school calculus teachers who are interested in applying to be a Reader at a future AP Reading can complete and submit an online application via AP Central (apcentral.collegeboard.com/readers) or request more information by e-mailing apreader@ets.org.

In June 2008, nearly 900 calculus teachers and mathematics professors gathered at the Kansas City Convention Center in Missouri to participate in the scoring session for the AP Calculus Exams. Some of the most experienced members of this group were asked to serve as Exam Leaders, Question Leaders, Question Team members, and Table Leaders, and they arrived at the Reading early to help prepare for the scoring session. The remaining Readers were divided into groups, with each group advised and supervised by Table Leaders. Under the guidance of the Chief Reader, Exam Leaders, Question Leaders, and selected Table Leaders assisted in establishing scoring guidelines, selecting sample student responses that exemplified the guidelines, and preparing for Reader training. All of the free-response questions on the 2008 AP Calculus AB and Calculus BC Exams were evaluated by the Readers at this single, central scoring session under the supervision of the Chief Reader.

Ensuring Accuracy

The primary goal of the scoring process is to have each Reader score responses fairly, consistently, and with the same guidelines as the other Readers. This goal is achieved through the careful application of detailed scoring guidelines, the thorough training of all Readers, and the various checks and balances that are applied throughout the AP Reading.

How the Scoring Guidelines Are Created

1. As the free-response questions are being developed and reviewed before the exams are given, the Development Committee and the Chief Reader discuss the scoring of the questions to ensure that the questions can be scored validly and reliably. The committee members provide preliminary guidance regarding the philosophy to be used in scoring the various free-response questions. The Chief Reader produces a rough draft of the scoring guidelines for each free-response question for committee input.
2. During the pre-Reading period, several important tasks are completed:
   - First, the Chief Reader produces a draft of the scoring guidelines for each free-response question. Exam Leaders and Question Leaders receive copies of the scoring guidelines and a set of actual student responses selected by ETS Assessment Specialists. The Exam Leaders and Question Leaders come to the Reading site prepared to discuss revisions to the scoring guidelines based on the review of student responses.
   - The Chief Reader, Exam Leaders, Question Leaders, Table Leaders, and ETS Assessment Specialists meet at the Reading site to discuss, review, and revise the draft scoring guidelines. Each Question Leader works with two Table Leaders who serve as Question Team members. The scoring guidelines are tested by applying them to actual student responses to the questions. The scoring guidelines are revised and adjusted, if necessary, to reflect not only the committee's original intent but also the full range of actual responses that will be encountered by the Readers.
3. Once the scoring of student responses begins, no changes are made to the scoring guidelines. Given the expertise of the Chief Reader and the analysis of many student responses by Exam Leaders, Question Leaders, Question Team members, and other Table Leaders in the pre-Reading period, these guidelines can be used to cover the whole range of student responses. During the first day of scoring of a particular question, Exam Leaders, Question Leaders, Question Team members, and Table Leaders ensure that everyone evaluating responses for that

question understands the scoring guidelines and can apply them reliably and fairly.

Training Readers to Apply the Scoring Guidelines

Because Reader training is so vital in ensuring that students receive an AP grade that accurately reflects their performance, the process is thorough:

1. On the first day of the Reading, the Chief Reader provides an overview of the exams and the scoring process to the entire group of Readers. At the Calculus Reading, Readers typically score four different free-response questions during the course of the Reading. These are usually two questions that are common to the Calculus AB and Calculus BC Exams, plus two other questions from either the Calculus AB Exam or the Calculus BC Exam. All Readers typically start with one of the common questions and receive training for scoring that question.
2. Each Question Leader directs a discussion of the question for which he or she is responsible, commenting on the requirements of the question and the expectations for student performance. The scoring guidelines for the question are explained and discussed.
3. The Readers are trained to apply the scoring guidelines by reading and evaluating samples of student responses that were selected at the pre-Reading session as clear examples of the various score points and as the kinds of responses that Readers are likely to encounter. Question and Table Leaders explain why the responses received particular scores and discuss the issues encountered in the sample responses.
4. Once the Readers understand the scoring guidelines and can apply them uniformly, the scoring of student responses begins. Experienced Readers are paired as "table partners" with less experienced Readers.
5. Table Leaders evaluate the responses scored by each Reader early in the process to help ensure that Readers are applying the scoring guidelines correctly.
6. Throughout the course of the Reading, Readers can discuss with their table partner or Table Leader any student response that presents a scoring difficulty. When necessary, the Question Leaders, Exam Leaders, or the Chief Reader are consulted. A student response that is problematic receives multiple readings and evaluations.

Maintaining the Scoring Guidelines

Throughout the Reading, Table Leaders continue to reinforce the use of the scoring guidelines by asking Readers to review sample responses that have already been discussed as clear examples of particular scores, or to score new samples that Table Leaders then discuss with their groups to make sure that the Readers have scored these samples fairly. This procedure helps Readers adhere to the standards of the group and helps to ensure that a student response will get the same score whether it is evaluated at the beginning, middle, or end of the Reading.

A potential problem is that a Reader could unintentionally score a student response higher or lower than it deserves because that same student performed well or poorly on other questions. The following steps are taken to prevent this so-called halo effect:

- A different Reader scores each question and the student's identity is unknown to the Reader. Thus, each Reader can evaluate student responses without being prejudiced by knowledge about individual students.
- No marks of any kind are made on the students' papers. Readers record the scores on a form that is identified only by the student's AP number. Readers are unable to see the scores that have been given to other responses in the exam booklet.

Here are some other methods that help ensure that everyone is adhering closely to the scoring guidelines:

- Table Leaders backread (reread) a portion of the student responses from each of the Readers in that Leader's group. This approach allows Table Leaders to guide their Readers toward appropriate and consistent interpretations of the scoring guidelines.
- Readers are paired, so that every Reader has a partner with whom to check for consistency and discuss problem cases; Table Leaders are also paired to help each other on questionable calls.
- Table Leaders randomly read selected responses to check for scoring consistency.
- Reliability data are periodically collected. This process involves Readers unknowingly rescoring student responses; the scores of the first and second readings are then compared to ensure that Readers are scoring consistently.

Preparing Students for the Exams

The AP Calculus courses (Calculus AB and Calculus BC) consist of a full high school academic year of work that is comparable to calculus courses in colleges and universities. The Calculus AB and Calculus BC courses have significant overlap, with Calculus BC also including additional topics in differential and integral calculus plus the topics of polynomial approximations and series. Calculus BC is an extension of Calculus AB rather than an enhancement; common topics require a similar depth of understanding. Both courses are intended to be challenging and demanding.

The *AP Calculus Course Description* includes a philosophy statement and goals for the AP Calculus courses and mathematical prerequisities for students studying calculus. As teachers focus on these goals, they will not only be preparing their students for the AP Calculus Exams; they will be preparing them for future study and applications of mathematics. It is assumed that most students preparing to take an AP Calculus Exam have completed a course in calculus that included instruction in the content areas outlined in the *AP Calculus Course Description*. Students who take an AP Calculus Exam are expected to demonstrate competence in calculus concepts and techniques.

Because the AP Calculus courses emphasize a multi-representational approach, students should be able to work with functions represented in a variety of ways—graphical, numerical, analytical, or verbal—and understand the connections between them. Students need to practice mathematical writing skills to help communicate their reasoning and explanations in the free-response portions of the exams. Students should have experience with justifying conclusions using calculus arguments. The exams have moved away from rote manipulation and toward questions that probe understanding of fundamental calculus concepts.

Students should be comfortable using a graphing calculator, particularly one with the four required capabilities—graphing a function within an arbitrary viewing window, finding zeros of functions (solving equations numerically), calculating derivatives numerically, and calculating definite integrals numerically. Because the exams have noncalculator parts, students are expected to know how to compute derivatives and antiderivatives of basic functions "by hand."

To do their best on the exams, students with the requisite calculus skills should become familiar with the format, timing, and particularly the free-response directions of the exams so that they will know what to expect when they take an AP Calculus Exam.

Teachers are encouraged to make use of the vast resources available at AP Central (apcentral.collegeboard.com), including the collection of free-response questions and scoring guidelines from previous AP Calculus Exams. Free-response questions are presented in a wide variety of contexts. Exam questions are intentionally written to be both nonrepetitive and nonpredictable from year to year.

Reviewing free-response questions from previous exams is a good practice to familiarize students with the style of questions, but teachers and students should not assume that the same topics, techniques, and contexts will be tested year after year.

Chapter II: The 2008 AP Calculus AB and Calculus BC Exams

- Exam Content and Format
- Giving a Practice Exam
- Instructions for Administering the Exam
- Blank Answer Sheet
- The Exams

Exam Content and Format

The 2008 AP Calculus AB and Calculus BC Exams are each 3 hours and 15 minutes in length and have two sections:

- A 105-minute multiple-choice section consisting of 45 questions accounting for 50 percent of the final grade. Part A consists of 28 questions in 55 minutes. Students are not allowed to use calculators for this part. Part B consists of 17 questions in 50 minutes. Students are required to have a graphing calculator for this part because some of the questions require the use of the calculator.
- A 90-minute free-response section consisting of 6 questions accounting for 50 percent of the final grade. Part A consists of 3 questions in 45 minutes. Students are required to have a graphing calculator for this part because some of the questions require the use of the calculator. Part B consists of 3 questions in 45 minutes. Students are not allowed to use calculators for this part. During the administration of Part B, students can work on the questions in Part A without the use of a calculator.

2008 AP Calculus AB and Calculus BC Exams Format

Multiple Choice (Section I)

Part A

28 questions ....55 minutes

No calculator allowed.

Part B

17 questions ....50 minutes

Graphing calculator required.

Free Response (Section II)

Part A

3 questions ....45 minutes

Graphing calculator required.

Part B

3 questions ....45 minutes

No calculator allowed.

During the timed portion for Part B, students may work on Part A questions without the use of a calculator.

Giving a Practice Exam

The following pages contain the instructions as they appeared in the 2008 *AP Examination Instructions* for administering the AP Calculus AB and Calculus BC Exams. Following the instructions are a blank 2008 answer sheet and the 2008 AP Calculus AB and Calculus BC Exams (Form Q). If you plan to use the released exams to test your students, you may wish to use these instructions to create an exam situation that closely resembles an actual administration. If so, read only the indented, boldface directions to the students; all other instructions are for the person administering the exam and need not be read aloud. Some instructions, such as those referring to the date, the time, and page numbers, are no longer relevant and should be ignored.

Another publication you might find useful is the *Packet of 10*—ten copies of either the 2008 AP Calculus AB Exam or the 2008 AP Calculus BC Exam, each with a blank answer sheet. You can order these titles online at the College Board Store (store.collegeboard.com).

Instructions for Administering the Exam
(from the 2008 *AP Examination Instructions* booklet)

The AP Program provides schools with scrambled multiple-choice sections for the regularly scheduled AP Calculus AB and AP Calculus BC Exams. You may seat students 4 feet apart, allowing for the testing of more students in fewer testing rooms. These exams have been designated with either form code 4EBP-Q or 4EBP-R on the multiple-choice section. All AP exam packets have serial numbers in the upper right hand corner of the multiple-choice section. For this exam, the range of serial numbers that each school receives will include both forms, packaged in sequential serial number order. The free-response sections are the same for all students.

For the scrambling to work, you must seat students so that the same form of the exam is not given to students seated next to each other. There should be at least 4 feet (approximately 1.2 meters) separating students. You must continue to be vigilant about opportunities for cheating. The sample seating plan for exams with scrambled multiple-choice sections on page 7 of the *AP Examination Instructions* booklet provides a snapshot of how students may be seated, the form codes, and a sample of the sequential serial numbers.

Graphing calculators are required to answer some of the questions on the AP Calculus Exams. Before starting the exam administration, make sure each student has a graphing calculator from the approved list on page 63 of the 2008 *AP Coordinator's Manual*. If a student does not have a graphing calculator from the approved list, you may provide one from your supply. If the student does not want to use the calculator you provide or does not want to use a calculator at all, he or she must sign the release statement on page 62 of the 2008 *AP Coordinator's Manual*.

During the administration of Section I, Part B, and Section II, Part A, students may have no more than two graphing calculators on their desks; calculators may not be shared. **Calculator memories do not need to be cleared before or after the exam.** Students with Hewlett-Packard 48–50 Series graphing calculators may use cards designed for use with these calculators. Proctors should make sure infrared ports (Hewlett-Packard) are not facing each other. Since graphing calculators can be used to store data, including text, proctors should monitor that students are using their calculators appropriately. Attempts by students to use the calculator to remove exam questions and/or answers from the room may result in the invalidation of AP Exam grades.

The AP Calculus AB Exam and the AP Calculus BC Exam should be administered simultaneously. They may be administered in separate rooms, or in the same room if it is more convenient.

These exams include survey questions. The time allowed for the survey questions is in addition to the actual test-taking time.

SECTION I: Multiple-Choice Questions

! **Do not begin the exam instructions below until you have completed the appropriate General Instructions for your group.**

Make sure you begin the exams at the designated time. When you have completed the General Instructions, say:

> **It is Wednesday morning, May 7, and you will be taking either the AP Calculus AB Exam or the AP Calculus BC Exam. In a moment, you will open the packet that contains your exam materials.**

> **By opening this packet, you agree to all of the AP Program's policies and procedures outlined in the 2007-08 *Bulletin for AP Students and Parents*. Please check to make sure you have the correct exam: Calculus AB or Calculus BC. Raise your hand if you do not have the correct exam. . . .**
>
> **You may now open your exam packet and take out the Section I booklet, but do not open the booklet or the shrinkwrapped Section II materials. Put the white seals aside. Read the statements on the front cover of Section I and look up when you have finished. . . .**
>
> **Now sign your name and write today's date. Look up when you have finished. . . .**
>
> **Now print your full legal name where indicated. Are there any questions? . . .**

Answer any questions. Then say:

> **Now turn to the back cover and read it completely. Look up when you have finished. . . .**
>
> **Are there any questions? . . .**

Answer any questions. Then say:

> **Section I is the multiple-choice portion of the exam. You may never discuss these specific multiple-choice questions at any time in any form with anyone, including your teacher and other students. If you disclose these questions through any means, your AP Exam grade will be canceled. Are there any questions? . . .**

Answer any questions. Then say:

> **You must complete the answer sheet using a No. 2 pencil only. Mark all of your responses on your answer sheet, one response per question. Completely fill in the ovals. There are more answer ovals on the answer sheet than there are questions, so you will have unused ovals when you reach the end. Your answer sheet will be scored by machine; any stray marks or smudges could be read as answers. If you need to erase, do so carefully and completely. No credit will be given for anything written in the exam booklet. Scratch paper is not allowed, but you may use the margins or any blank space in the exam booklet for scratch work.**
>
> **Section I is divided into two parts. Each part is timed separately, and you may work on each part only during the time allotted for it. Calculators are not allowed in Part A. Please put your calculators under your chair. Are there any questions? . . .**

Answer all questions regarding procedure. Then say:

> **You have 55 minutes for Part A. Part A questions are numbered 1 through 28. Mark your responses for these questions on page 2 of your answer sheet. Open your Section I booklet and begin.**

Note Start Time here __________. Note Stop Time here __________. You and your proctors should make sure students are marking their answers in pencil on page 2 of their answer sheets and that they are not looking beyond Part A. The line of A's at the top of each page will assist you in monitoring students' work. After 55 minutes, say:

Stop working on Part A and turn to page 22 for Calculus AB Form Q, page 20 for Calculus AB Form R, and page 20 for Calculus BC (both Form Q and Form R) in your Section I booklet. . . .

On page 20 or 22, you should see an area marked "PLACE SEAL HERE." Making sure all of your other exam materials, including your answer sheet, are out of the way, take one of your seals and press it on that area and then fold the seal over the open edge to the front cover. Be sure you don't seal the Part B section of the booklet or let the seal touch anything except the marked areas.

After all students have sealed Part A, say:

Graphing calculators are required for Part B. You may get your calculators from under your chair and place them on your desk. Part B questions are numbered 76 through 92. Fold your answer sheet so only page 3 is showing and mark your responses for these questions on that page. You have 50 minutes for Part B. You may begin.

Note Start Time here __________. Note Stop Time here __________. You and your proctors should walk around and make sure students have sealed their booklets properly and are now working on Part B. The large B's in an alternating shaded pattern at the top of each page will assist you in monitoring their work. Proctors should make sure that students are using their calculators appropriately. Proctors should also make sure Hewlett-Packard calculators' infrared ports are not facing each other. After 50 minutes, say:

Stop working and turn to page 37 for Calculus AB Form Q, page 35 for Calculus AB Form R, and page 33 for Calculus BC. You have 3 minutes to answer Questions 93–96. These are survey questions and will not affect your grade. You may not go back to work on any of the exam questions. . . .

Give students approximately 3 minutes to answer the survey questions. Then say:

Close your booklet and put your answer sheet on your desk, face up, with the fold to your left. I will now collect your answer sheet.

After you have collected an answer sheet from each student, say:

Take your seals and press one on each area of your exam booklet marked "PLACE SEAL HERE." Fold them over the open edges and press them to the back cover. When you have finished, place the booklet on your desk with the cover face up and the fold to your left. . . .

I will now collect your Section I booklet.

As you collect the sealed Section I booklets, check to be sure that each student has signed the front cover. There is a 10-minute break between Sections I and II. When all Section I materials have been collected and accounted for and you are ready for the break, say:

Please listen carefully to these instructions before we take a break. Everything you placed under your chair at the beginning of the exam must remain there. You are not allowed to consult teachers, other students, or textbooks about the exam materials during the break. You may not make phone calls, send text messages, check e-mail, access a computer, calculator, cell phone, PDA, MP3 player, e-mail/messaging device, or any other electronic or communication device. Remember, you are not allowed to discuss the multiple-choice section of this exam with <u>anyone at any time</u>. Failure to

adhere to any of these rules could result in invalidation of your grade. Please leave your shrinkwrapped Section II package on top of your desk during the break. You may get up, talk, go to the restroom, or get a drink. Are there any questions? . . .

Answer all questions regarding procedure. Then say:

Let's begin our break. Testing will resume at __________.

SECTION II: Free-Response Questions

After the break, say:

May I have everyone's attention? Place your Student Pack on your desk. . . .

You may now open the shrinkwrapped Section II package. . . .

Read the bulleted statements on the front cover of the pink booklet. Look up when you have finished. . . .

Now place an AP number label on the shaded box. If you don't place an AP number label on this box, it may be impossible to identify your booklet, which could delay or jeopardize your AP grade. If you don't have any AP number labels, write your AP number in the box. Look up when you have finished. . . .

Read the last statement. . . .

Using a pen with black or dark blue ink, print the first, middle, and last initials of your legal name in the boxes and print today's date where indicated. This constitutes your signature and your agreement to the conditions stated on the front cover. . . .

Turn to the back cover and read Item 1 under "Important Identification Information." Print your identification information in the boxes. Note that you must print the first two letters of your LAST name and the first letter of your FIRST name. Look up when you have finished. . . .

In Item 2, print your date of birth in the boxes. . . .

Read Item 3 and copy the school code you printed on the front of your Student Pack into the boxes. . . .

Read Item 4. . . .

Are there any questions? . . .

Answer all questions regarding procedure. Then say:

I need to collect the Student Pack from anyone who will be taking another AP Exam. If you are taking another AP Exam, put your Student Pack on your desk. You may keep it only if you are not taking any other AP Exams this year. If you have no other AP Exams to take, place your Student Pack under your chair now. . . .

While Student Packs are being collected, read the "At a Glance" column and the instructions on the back cover of the pink booklet, paying careful attention to the bulleted statements. Do not open the pink booklet or break the seal on the blue insert until you are told to do so. Look up when you have finished. . . .

Collect the Student Packs. Then say:

Are there any questions? . . .

Answer all questions regarding procedure. Then say:

Now open the Section II booklet and tear out the green insert that is in the center of the booklet. In the upper right-hand corner of the cover, print your name, your teacher's name, and your school's name. . . .

Read the information on the front cover of the green insert. Look up when you have finished. . . .

Section II also has two parts that are timed separately. You are responsible for pacing yourself, and may proceed freely from one question to the next within each part. Graphing calculators are required for Part A, so you may keep your calculators on your desk. You may use the green insert for scratch paper but you MUST write your answers in the appropriate space in the pink booklet using a No. 2 pencil or a pen with black or dark blue ink. Put the sealed blue insert aside; you will need it for Part B. Are there any questions? . . .

Answer any questions. Then say:

You have 45 minutes to answer the questions in Part A. If you need more paper during the exam, raise your hand. At the top of each extra piece of paper you use, be sure to write your AP number and the number of the question you are working on. Open the green insert and begin.

Note Start Time here __________. Note Stop Time here __________. You and your proctors should make sure students are working on Part A and writing their answers in their pink booklets using pencils or pens with black or dark blue ink. The pages for Part A are marked with large 1's, 2's, and 3's at the top of each page to assist you in monitoring their work. After 35 minutes, say:

There are 10 minutes remaining in Part A.

After 10 minutes, say:

Stop working on Part A. Calculators are not allowed for Part B. Please put all of your calculators under your chair. . . .

You will use the blue insert for Part B. Take your blue insert and print your name, your teacher's name, and your school's name in the upper right-hand corner of the cover. . . .

You have 45 minutes for Part B. During this time you may go back to Part A, but you may not use your calculators. If you go back to Part A, you may use the green insert. Remember to write your answer to each part of each problem in the appropriate space in the pink booklet. Are there any questions? . . .

Answer any questions. Then say:

Using your finger, break open the seals on the blue insert. Do not peel the seals away from the booklet. You may begin Part B. . . .

Note Start Time here __________. Note Stop Time here __________. After 35 minutes, say:

There are 10 minutes remaining in Part B.

After 10 minutes, say:

> **Stop working and close your exam booklet and the inserts. Put your pink booklet on your desk, face up, with the fold to your left. Put your green and your blue inserts next to it. Remain in your seat, without talking, while the exam materials are collected. . . .**

Collect a pink Section II booklet, a green insert, and a blue insert from every student. **Check the front cover of each pink booklet to ensure that the student has placed an AP number label on the shaded box and printed his or her initials and today's date. Check that the student has completed the "Important Identification Information" area on the back cover, and that answers have been written in the pink booklet and not in the green or the blue inserts.** The green and the blue inserts must be stored securely for no fewer than two school days. After the two-day holding time, the inserts may be given to the appropriate AP teacher(s) for return to the students. When all exam materials have been collected and accounted for, say:

> **Your teacher will return your green and your blue inserts to you in about two days. You may not discuss the free-response questions with anyone until that time. Remember that the multiple-choice questions may never be discussed or shared in any way at any time. You should receive your grade reports in the mail about the third week of July. You are now dismissed.**

Exam materials should be put in locked storage until they are returned to the AP Program after your school's last administration. Before storing materials, check your list of students who are eligible for fee reductions and fill in the appropriate oval on their registration answer sheets. To receive a separate *AP Instructional Planning Report* or student grade roster for each AP class taught, fill in the appropriate oval in the "School Use Only" section of the answer sheet. See "Post-Exam Activities" in the 2008 *AP Coordinator's Manual.*

USE PENCIL ONLY FOR THE ENTIRE ANSWER SHEET.

CollegeBoard AP Answer Sheet for May 2008, Form 4EBP PAGE 1

NAME AND EXAM AREA – COMPLETE THIS AREA AT EVERY EXAM.

To maintain the security of the exam and the validity of my AP grade, I will allow no one else to see the multiple-choice questions. I will seal the multiple-choice booklet when asked to do so, and I will not discuss these questions with anyone at any time after the completion of the section. I am aware of and agree to the AP Program's policies and procedures as outlined in the 2007-08 *Bulletin for AP Students and Parents*, including using testing accommodations (e.g., extended time, computer, etc.) only if I have been preapproved by College Board Services for Students with Disabilities.

A. SIGNATURE ______________________________

Sign your legal name as it will appear on your college applications.

PLACE YOUR AP® NUMBER LABEL OR WRITE YOUR AP NUMBER HERE AT EVERY EXAM.

__ __ __ __ __ __ __ __

B. LEGAL NAME Omit apostrophes, Jr., or II.

Legal Last Name—first 15 letters | Legal First Name—first 12 letters | MI

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C. YOUR AP NUMBER

⓪ ① ② ③ ④ ⑤ ⑥ ⑦ ⑧ ⑨

D. ADMIN. DAY IN MAY

⓪ ① ② ③ | ⓪ ① ② ③ ④ ⑤ ⑥ ⑦ ⑧ ⑨

E. TIME OF DAY

○ 6 AM
○ 7 AM
○ 8 AM
○ 9 AM
○ 10 AM
○ 11 AM
○ 12 PM
○ 1 PM
○ 2 PM
○ 3 PM
○ 4 PM
○ 5 PM

F. AP EXAM I AM TAKING USING THIS ANSWER SHEET

Print exam name: ______________________ Print form code (e.g., 4EBP-R) from M-C booklet: ____________

Fill in the oval below that corresponds to the exam that you are taking.

07 ○ U.S. History 4EBP-Q	43 ○ European History	78 ○ Physics B
07 ○ U.S. History 4EBP-R	48 ○ French Language	80 ○ Physics C: Mech.
13 ○ Art History	51 ○ French Literature	82 ○ Physics C: E & M
14 ○ Art: Studio Drawing	53 ○ Geography: Human	85 ○ Psychology
15 ○ Art: Studio 2-D Design	55 ○ German Language	87 ○ Spanish Language
16 ○ Art: Studio 3-D Design	57 ○ Gov. & Pol.: U.S.	89 ○ Spanish Literature
20 ○ Biology	58 ○ Gov. & Pol.: Comp.	90 ○ Statistics
25 ○ Chemistry	60 ○ Latin: Vergil	93 ○ World History
28 ○ Chinese Lang. & Culture	61 ○ Latin Literature	
31 ○ Computer Science A	62 ○ Italian Lang. & Culture	
33 ○ Computer Science AB	64 ○ Japanese Lang. & Culture	
34 ○ Economics: Micro	66 ○ Calculus AB 4EBP-Q	
35 ○ Economics: Macro	66 ○ Calculus AB 4EBP-R	
36 ○ Eng. Language & Comp.	68 ○ Calculus BC 4EBP-Q	
37 ○ Eng. Literature & Comp.	68 ○ Calculus BC 4EBP-R	
40 ○ Environmental Science	75 ○ Music Theory	

G. ONLINE PROVIDER CODE

⓪ ① ② ③ ④ ⑤ ⑥ ⑦ ⑧ ⑨

H. MULTIPLE-CHOICE BOOKLET SERIAL NUMBER

S

⓪ ① ② ③ ④ ⑤ ⑥ ⑦ ⑧ ⑨

743180

STUDENT INFORMATION AREA—COMPLETE THIS AREA ONLY ONCE.

I. SEX

① Male
② Female

J. CURRENT GRADE LEVEL

① Not yet in 9th grade
② 9th grade
③ 10th grade
④ 11th grade
⑤ 12th grade
⑥ No longer in high school

K. DATE OF BIRTH

Month	Day	Year
01 ○ Jan.		
02 ○ Feb.		
03 ○ Mar.	⓪ ⓪	⓪ ⓪
04 ○ Apr.	① ①	① ①
05 ○ May	② ②	② ②
06 ○ June	③ ③	③ ③
07 ○ July	④	④ ④
08 ○ Aug.	⑤	⑤ ⑤
09 ○ Sept.	⑥	⑥ ⑥
10 ○ Oct.	⑦	⑦ ⑦
11 ○ Nov.	⑧	⑧ ⑧
12 ○ Dec.	⑨	⑨ ⑨

L. SOCIAL SECURITY NUMBER (Optional, but preferred)

⓪ ① ② ③ ④ ⑤ ⑥ ⑦ ⑧ ⑨

M. ETHNICITY/RACE

① American Indian or Alaska Native
② Asian, Asian American, or Pacific Islander
③ Black or African American
④ Mexican or Mexican American
⑤ Puerto Rican
⑥ Other Hispanic, Latino, or Latin American
⑦ White
⑧ Other

N. EXPECTED DATE OF COLLEGE ENTRANCE

Ⓕ Fall	⓪ 2008
Ⓦ Winter/Spring	① 2009
Ⓢ Summer	② 2010
Ⓤ Undecided	③ 2011

O. WHAT LANGUAGE DO YOU KNOW BEST?

① English
② English and another language about the same
③ Another language

P. Complete ONLY if you are a SOPHOMORE or a JUNIOR.

I want the College Board to send information about me to colleges, universities, and government scholarship programs interested in students like me.

○ Yes ○ No

Q. PARENTAL EDUCATION LEVEL

Father / Male Guardian	Mother / Female Guardian	
○	○	Grade school
○	○	Some high school
○	○	High school diploma or equivalent
○	○	Business or trade school
○	○	Some college
○	○	Associate or two-year degree
○	○	Bachelor's or four-year degree
○	○	Some graduate or professional school
○	○	Graduate or professional degree

SCHOOL USE ONLY

Instructional Planning Report/Student Grade Roster

① ② ③ ④ ⑤ ⑥ ⑦ ⑧ ⑨

Fee Reduction Granted

○ Option 1 ○ Option 2

ETS USE ONLY

Exam ① ② | Exam ① ②

⓪ ① ② ③ ④ ⑤ ⑥ ⑦ ⑧ ⑨

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CollegeBoard AP

R. This section is for the survey questions in the AP Student Pack. **(Do not put responses to exam questions in this section.)** Be sure each mark is dark and completely fills the oval.

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Do not complete this section unless instructed to do so.

S. If this answer sheet is for the Chinese Language and Culture, French Language, French Literature, German Language, Italian Language and Culture, Japanese Language and Culture, Spanish Language, or Spanish Literature Exam, please answer the following questions. (Your responses will not affect your grade.)

1. Have you lived or studied for one month or more in a country where the language of the exam you are now taking is spoken? ○ Yes ○ No
2. Do you regularly speak or hear the language at home? ○ Yes ○ No

Indicate your answers to the exam questions in this section. If a question has only four answer options, do not mark option E. Your answer sheet will be scored by machine. Use only No. 2 pencils to mark your answers on pages 2 and 3 (one response per question). After you have determined your response, be sure to completely fill in the oval corresponding to the number of the question you are answering. Stray marks and smudges could be read as answers, so erase carefully and completely. Any improper gridding may affect your grade. Answers written in the multiple-choice booklet will not be scored.

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FOR QUESTIONS 76-151, SEE PAGE 3.

DO NOT WRITE IN THIS AREA.

CollegeBoard AP

PAGE 3

Be sure each mark is dark and completely fills the oval. If a question has only four answer options, do not mark option E.

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PT2			
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EQ			
TA1			
TA2			

DO NOT WRITE IN THIS AREA.

HOME ADDRESS AND SCHOOL AREA—COMPLETE THIS AREA ONLY ONCE.

T. YOUR MAILING ADDRESS

- Your grade report will be mailed to this address in July.
- Using the abbreviations given in your AP Student Pack, print your address in the boxes below. If your international address does not fit, see item V, below.
- Indicate a space in your address by leaving a blank box — do not grid that column. Only one oval is to be filled in for each column.

Street

City

STATE

1 ○ AL
2 ○ AK
3 ○ AZ
4 ○ AR
5 ○ CA
6 ○ CO
7 ○ CT
8 ○ DE
9 ○ DC
10 ○ FL
11 ○ GA
12 ○ HI
13 ○ ID
14 ○ IL
15 ○ IN
16 ○ IA
17 ○ KS
18 ○ KY
19 ○ LA
20 ○ ME
21 ○ MD
22 ○ MA
23 ○ MI
24 ○ MN
25 ○ MS
26 ○ MO
27 ○ MT
28 ○ NE
29 ○ NV
30 ○ NH
31 ○ NJ
32 ○ NM
33 ○ NY
34 ○ NC
35 ○ ND
36 ○ OH
37 ○ OK
38 ○ OR
39 ○ PA
40 ○ RI
41 ○ SC
42 ○ SD
43 ○ TN
44 ○ TX
45 ○ UT
46 ○ VT
47 ○ VA
48 ○ WA
49 ○ WV
50 ○ WI
51 ○ WY
52 ○ Puerto Rico
53 ○ Other

ZIP OR POSTAL CODE

U. COUNTRY CODE

W. TELEPHONE NUMBER

Area Code

INTERNATIONAL TELEPHONE

If your international telephone number is longer than 10 digits, write the entire number below.

FOR STUDENTS OUTSIDE THE UNITED STATES ONLY

V. If the address gridded above is not complete enough for delivery of your grade report, please fill in this oval and print your complete address below. ——→ ○

Address

Address

City

State or Province

Country

ZIP or Postal Code

X. SCHOOL YOU ATTEND

School Code

School Name, City, and State

← Make sure you have correctly entered your School Code, filled in the appropriate ovals, and completed the information below.

School Name

City

State

Y. COLLEGE TO RECEIVE YOUR AP GRADE REPORT

College Code

College Name and Address

Using the College Code list in the AP Student Pack, indicate the **one** college that you want to receive your AP Grade Report by writing in the college code number, gridding the appropriate ovals, and completing the information below.

College Name

City

State

AP® Calculus AB Exam

SECTION I: Multiple-Choice Questions

At a Glance

Total Time
1 hour, 45 minutes
Number of Questions
45
Percent of Total Grade
50%
Writing Instrument
Pencil required

Part A

Number of Questions
28
Time
55 minutes
Electronic Device
None allowed

Part B

Number of Questions
17
Time
50 minutes
Electronic Device
Graphing calculator required

Instructions

Section I of this exam contains 45 multiple-choice questions and 4 survey questions. For Part A, fill in only the ovals for numbers 1 through 28 on page 2 of the answer sheet. For Part B, fill in only the ovals for numbers 76 through 92 on page 3 of the answer sheet. The survey questions are numbers 93 through 96.

Indicate all of your answers to the multiple-choice questions on the answer sheet. No credit will be given for anything written in this exam booklet, but you may use the booklet for notes or scratch work. After you have decided which of the suggested answers is best, completely fill in the corresponding oval on the answer sheet. Give only one answer to each question. If you change an answer, be sure that the previous mark is erased completely. Here is a sample question and answer.

Sample Question	Sample Answer
Chicago is a (A) state (B) city (C) country (D) continent (E) village	Ⓐ ● Ⓒ Ⓓ Ⓔ

Use your time effectively, working as quickly as you can without losing accuracy. Do not spend too much time on any one question. Go on to other questions and come back to the ones you have not answered if you have time. It is not expected that everyone will know the answers to all of the multiple-choice questions.

About Guessing

Many students wonder whether or not to guess the answers to questions about which they are not certain. In this section of the exam, as a correction for random guessing, one-fourth of the number of questions you answer incorrectly will be subtracted from the number of questions you answer correctly. If you are not sure of the best answer but have some knowledge of the question and are able to eliminate one or more of the answer choices, your chance of answering correctly is improved, and it may be to your advantage to answer such a question.

CALCULUS AB
SECTION I, Part A
Time—55 minutes
Number of questions—28

A CALCULATOR MAY NOT BE USED ON THIS PART OF THE EXAM.

Directions: Solve each of the following problems, using the available space for scratch work. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding oval on the answer sheet. No credit will be given for anything written in the exam book. Do not spend too much time on any one problem.

In this exam:

(1) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

(2) The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1} x = \arcsin x$).

1. $\lim_{x\to\infty} \dfrac{(2x-1)(3-x)}{(x-1)(x+3)}$ is

(A) -3 (B) -2 (C) 2 (D) 3 (E) nonexistent

---

2. $\int \dfrac{1}{x^2}\, dx =$

(A) $\ln x^2 + C$ (B) $-\ln x^2 + C$ (C) $x^{-1} + C$ (D) $-x^{-1} + C$ (E) $-2x^{-3} + C$

3. If $f(x) = (x-1)\left(x^2+2\right)^3$, then $f'(x) =$

(A) $6x\left(x^2+2\right)^2$

(B) $6x(x-1)\left(x^2+2\right)^2$

(C) $\left(x^2+2\right)^2\left(x^2+3x-1\right)$

(D) $\left(x^2+2\right)^2\left(7x^2-6x+2\right)$

(E) $-3(x-1)\left(x^2+2\right)^2$

4. $\int(\sin(2x)+\cos(2x))\,dx =$

(A) $\frac{1}{2}\cos(2x)+\frac{1}{2}\sin(2x)+C$

(B) $-\frac{1}{2}\cos(2x)+\frac{1}{2}\sin(2x)+C$

(C) $2\cos(2x)+2\sin(2x)+C$

(D) $2\cos(2x)-2\sin(2x)+C$

(E) $-2\cos(2x)+2\sin(2x)+C$

5. $\lim\limits_{x \to 0} \dfrac{5x^4 + 8x^2}{3x^4 - 16x^2}$ is

(A) $-\dfrac{1}{2}$ (B) 0 (C) 1 (D) $\dfrac{5}{3}$ (E) nonexistent

---

$$f(x) = \begin{cases} \dfrac{x^2 - 4}{x - 2} & \text{if } x \neq 2 \\ 1 & \text{if } x = 2 \end{cases}$$

6. Let f be the function defined above. Which of the following statements about f are true?

I. f has a limit at $x = 2$.

II. f is continuous at $x = 2$.

III. f is differentiable at $x = 2$.

(A) I only

(B) II only

(C) III only

(D) I and II only

(E) I, II, and III

GO ON TO THE NEXT PAGE.

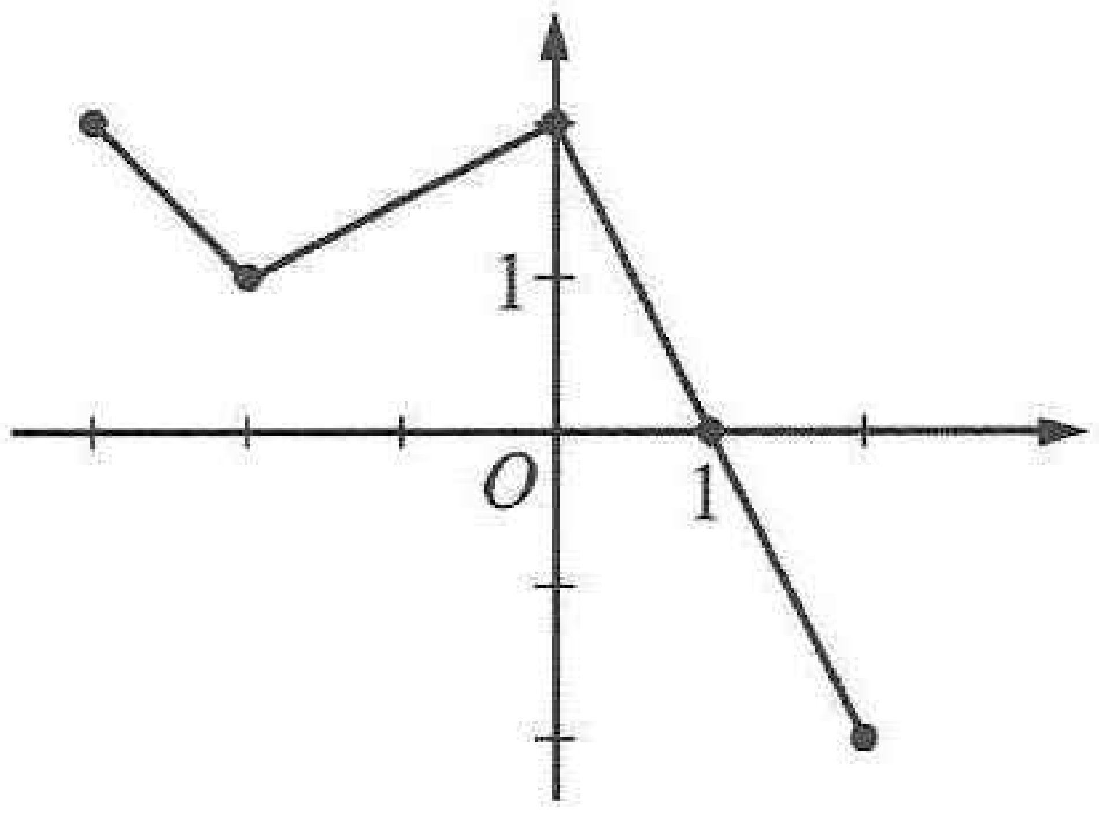
7. A particle moves along the x-axis with velocity given by $v(t) = 3t^2 + 6t$ for time $t \geq 0$. If the particle is at position $x = 2$ at time $t = 0$, what is the position of the particle at time $t = 1$?

(A) 4 (B) 6 (C) 9 (D) 11 (E) 12

---

8. If $f(x) = \cos(3x)$, then $f'\left(\frac{\pi}{9}\right) =$

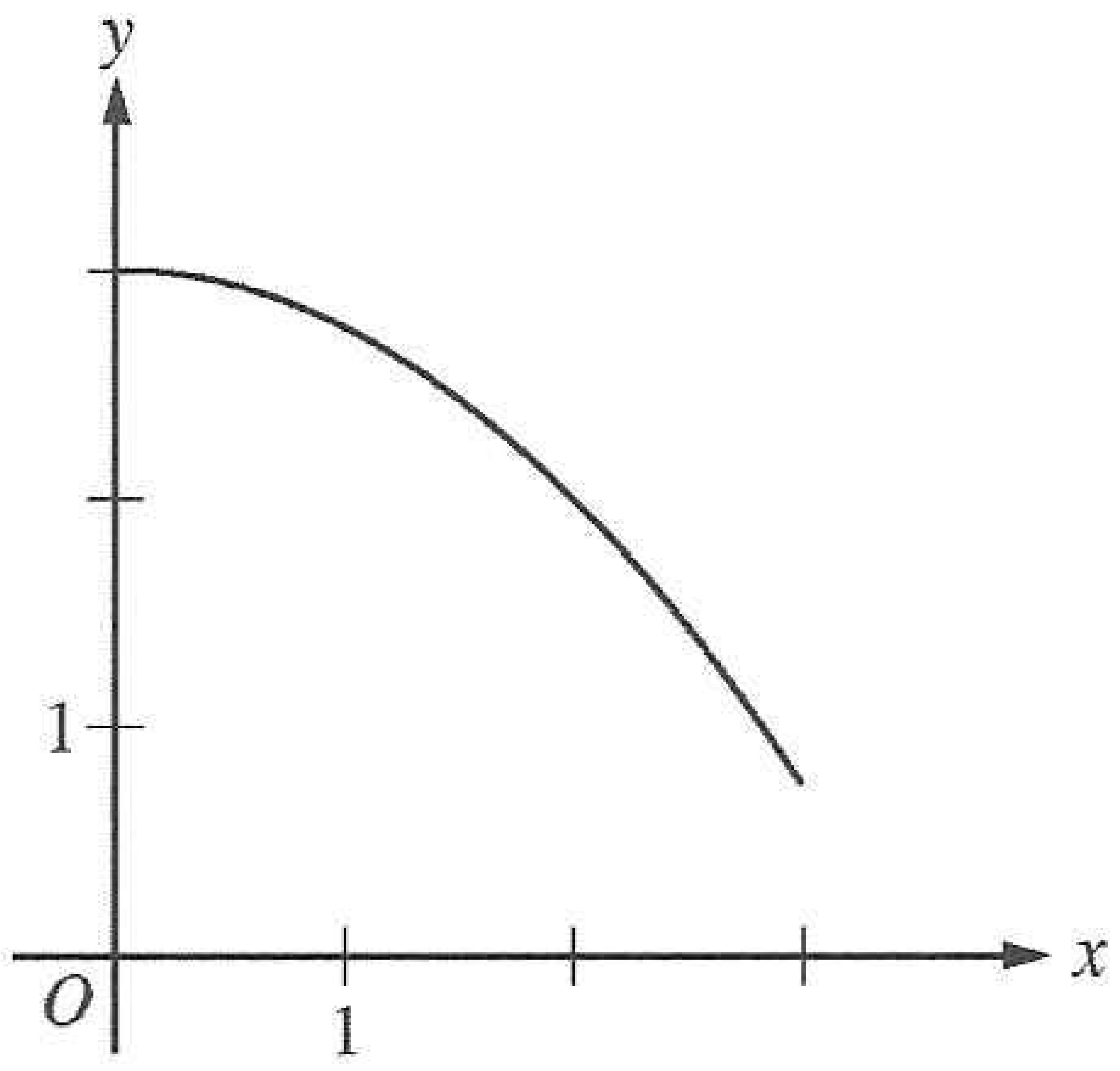
(A) $\frac{3\sqrt{3}}{2}$ (B) $\frac{\sqrt{3}}{2}$ (C) $-\frac{\sqrt{3}}{2}$ (D) $-\frac{3}{2}$ (E) $-\frac{3\sqrt{3}}{2}$



Graph of f

9. The graph of the piecewise linear function f is shown in the figure above. If $g(x) = \int_{-2}^{x} f(t)\,dt$, which of the following values is greatest?

(A) $g(-3)$ (B) $g(-2)$ (C) $g(0)$ (D) $g(1)$ (E) $g(2)$



Graph of f

10. The graph of the function f is shown above for $0 \leq x \leq 3$. Of the following, which has the least value?

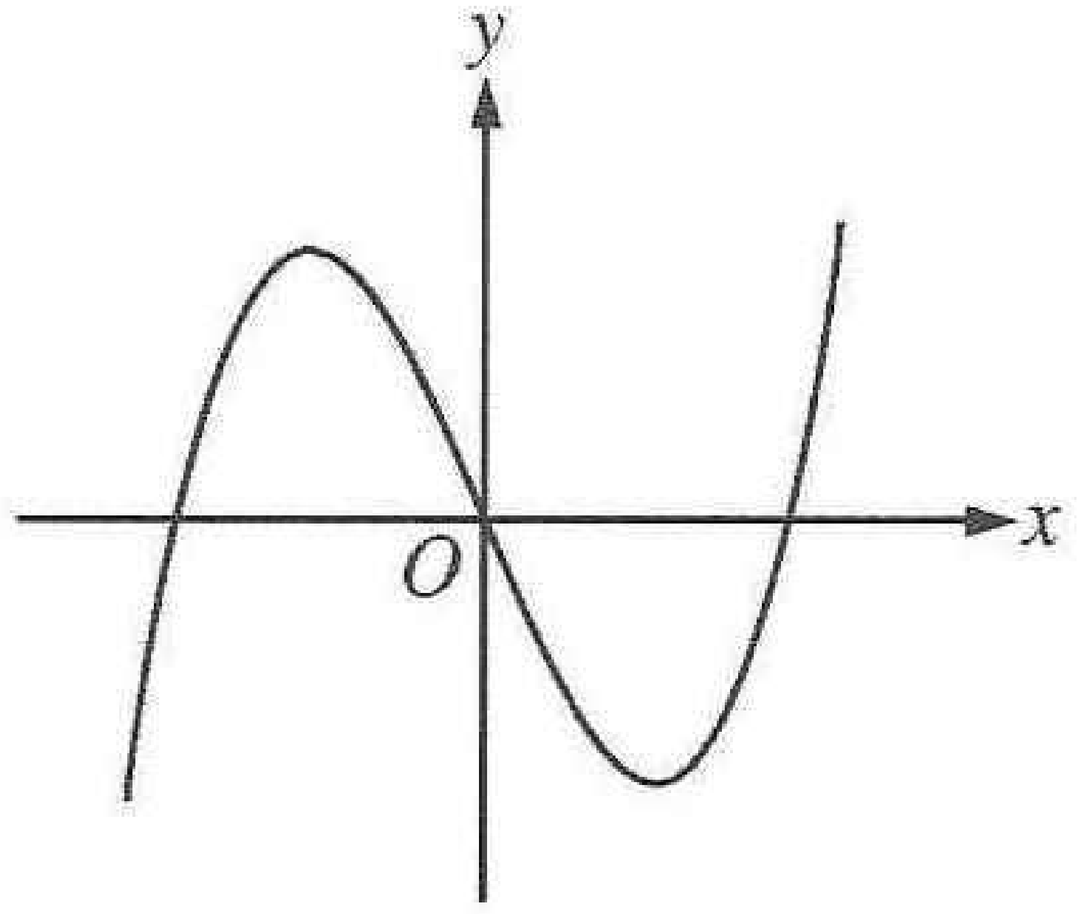
(A) $\int_1^3 f(x)\,dx$

(B) Left Riemann sum approximation of $\int_1^3 f(x)\,dx$ with 4 subintervals of equal length

(C) Right Riemann sum approximation of $\int_1^3 f(x)\,dx$ with 4 subintervals of equal length

(D) Midpoint Riemann sum approximation of $\int_1^3 f(x)\,dx$ with 4 subintervals of equal length

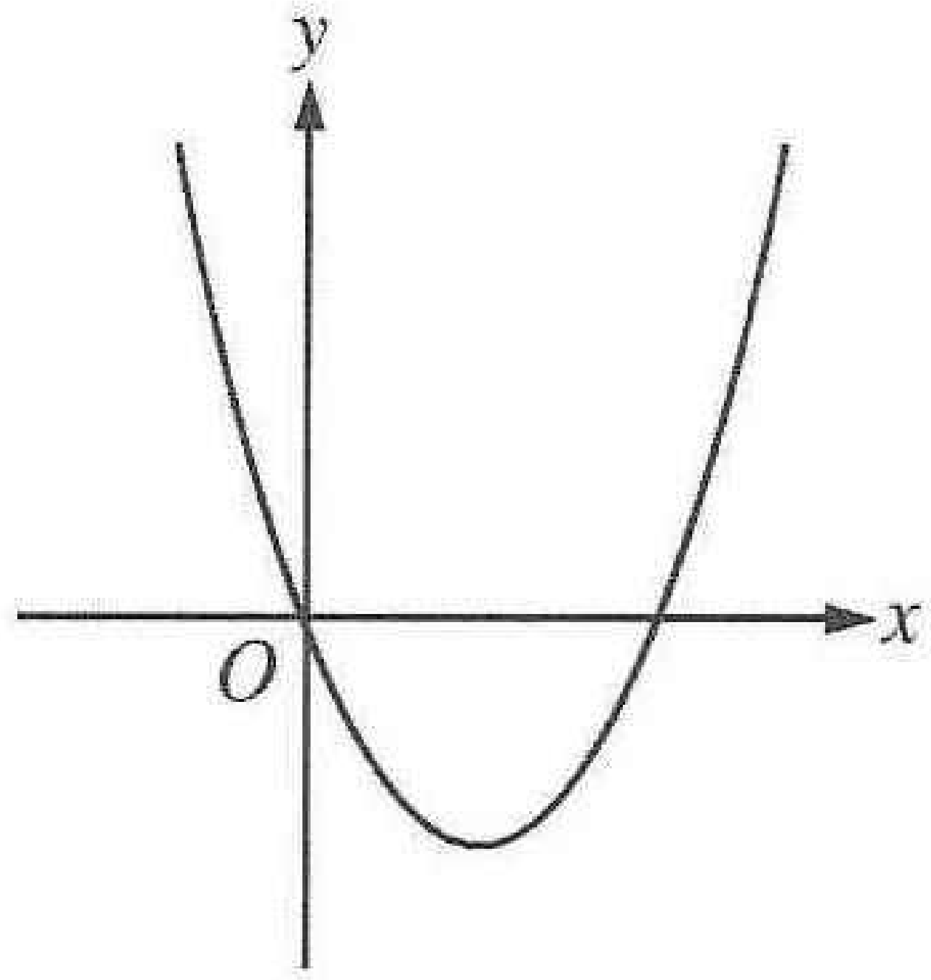
(E) Trapezoidal sum approximation of $\int_1^3 f(x)\,dx$ with 4 subintervals of equal length



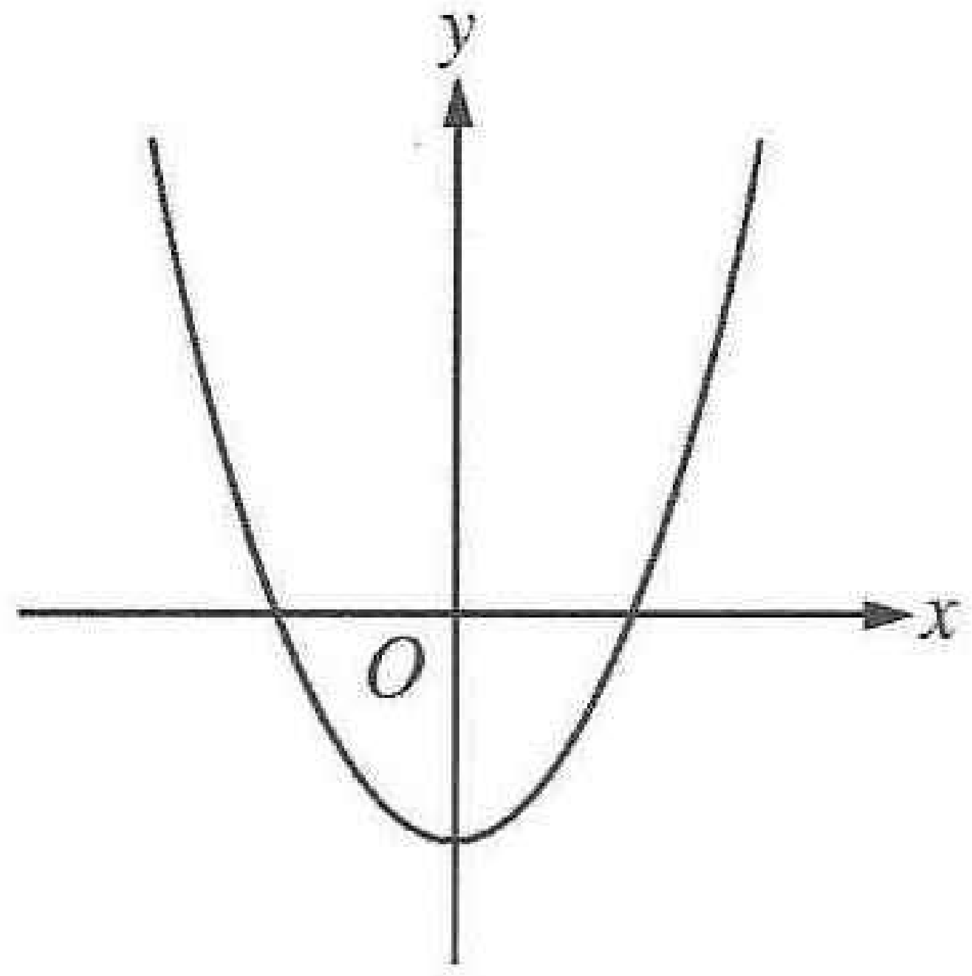
Graph of f

11. The graph of a function f is shown above. Which of the following could be the graph of f', the derivative of f?

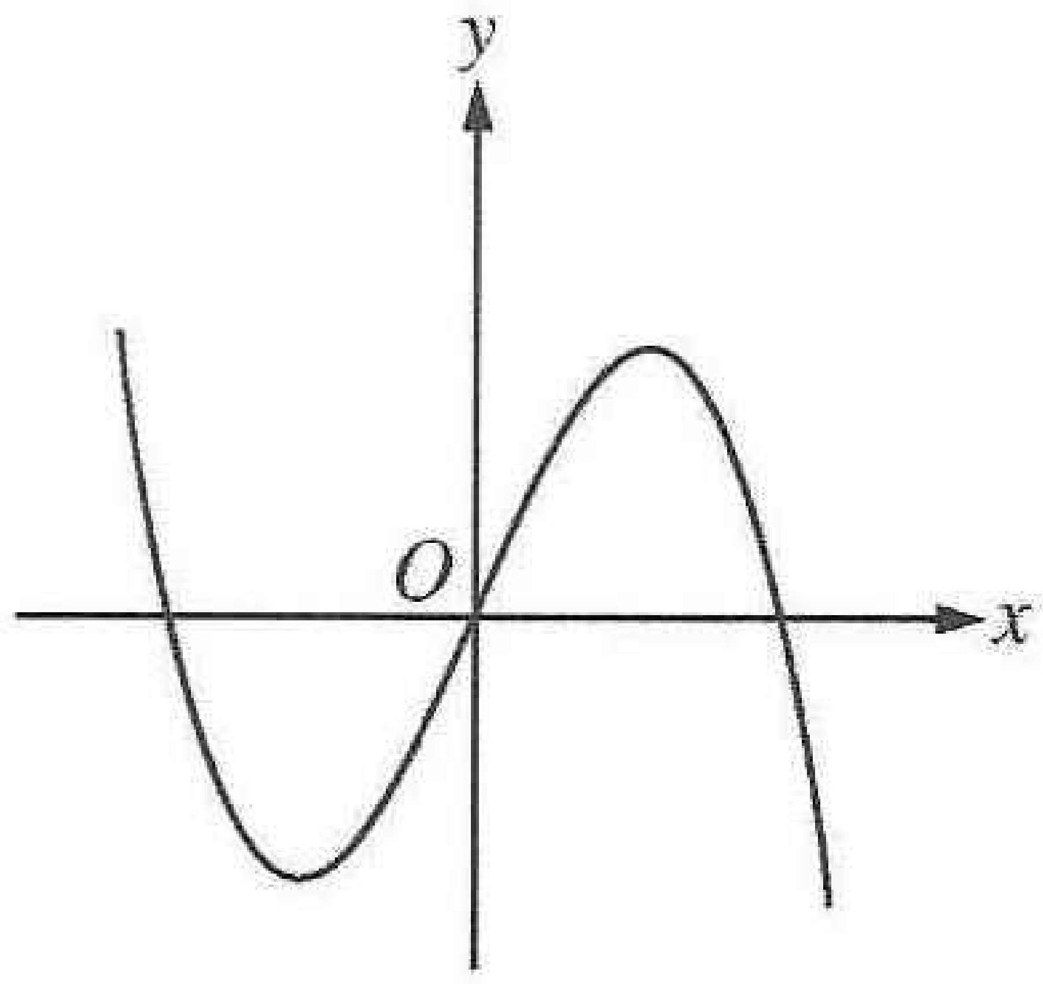
(A)



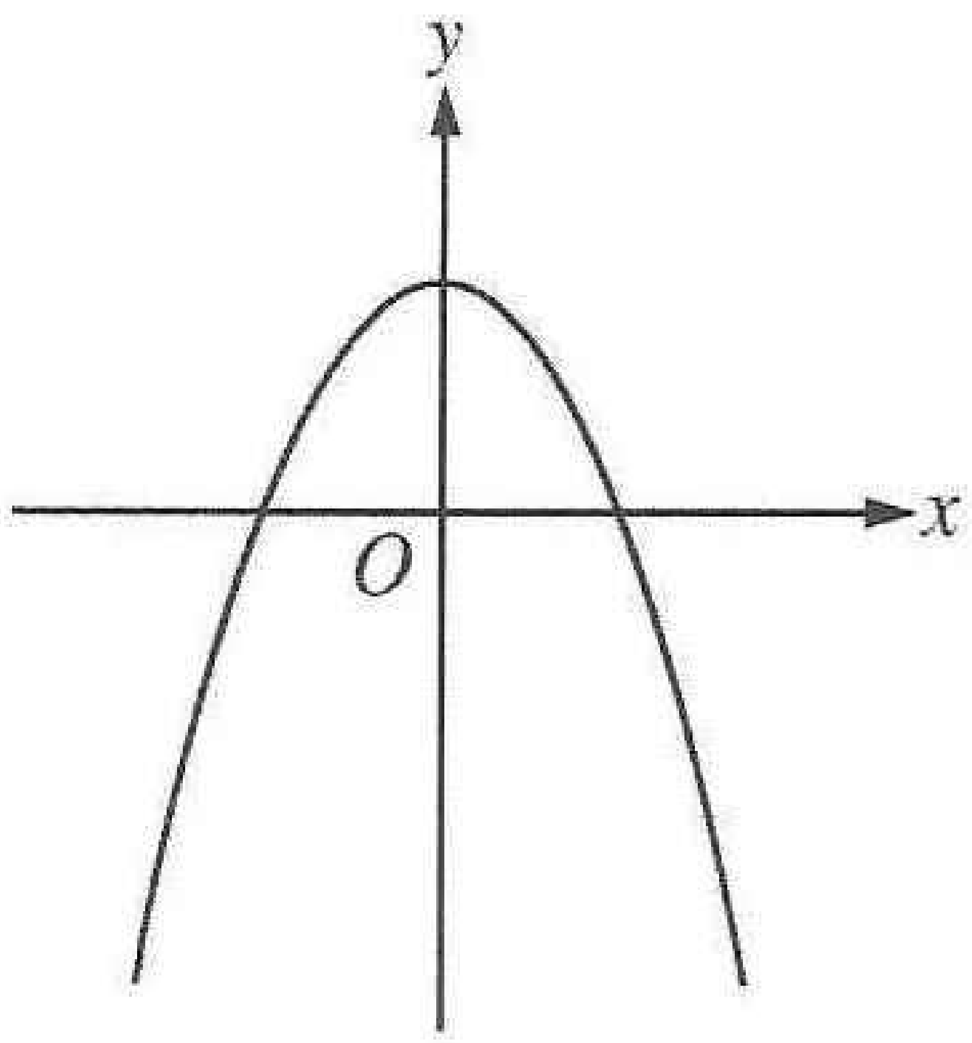
(B)



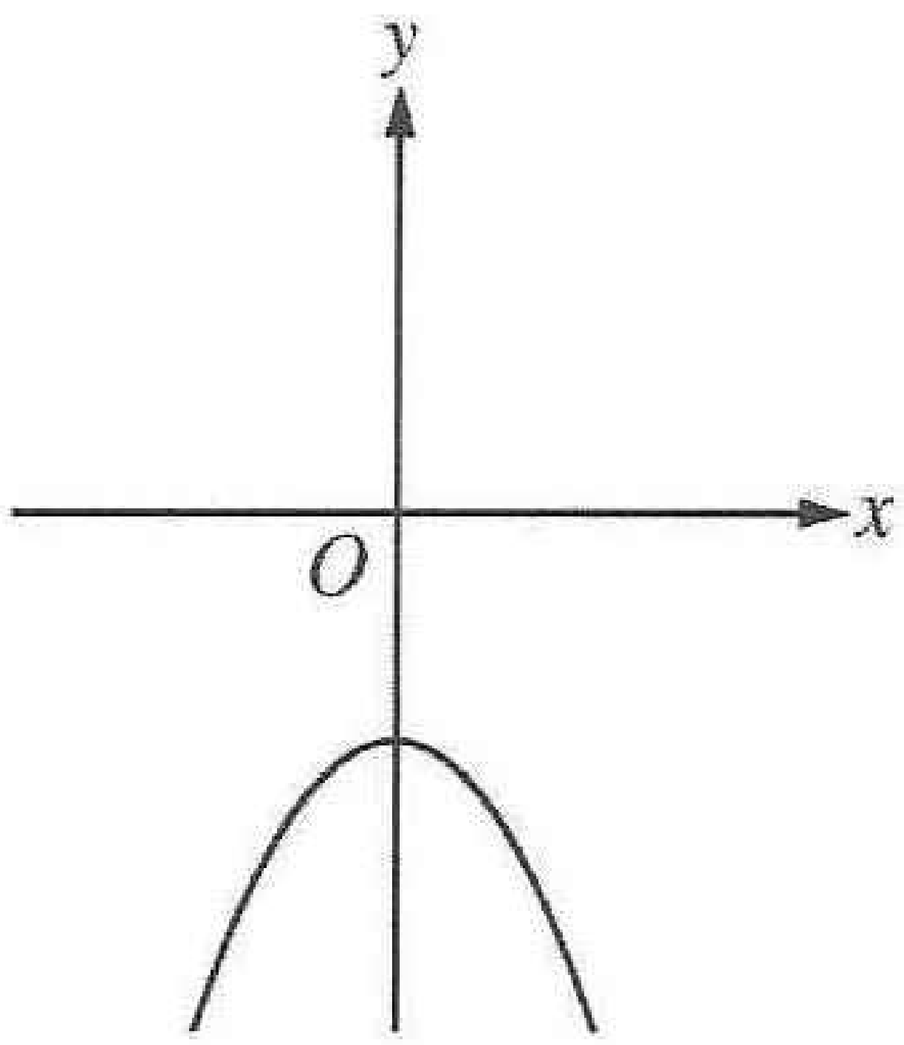
(C)



(D)



(E)



12. If $f(x) = e^{(2/x)}$, then $f'(x) =$

(A) $2e^{(2/x)} \ln x$ (B) $e^{(2/x)}$ (C) $e^{\left(-2/x^2\right)}$ (D) $-\frac{2}{x^2} e^{(2/x)}$ (E) $-2x^2 e^{(2/x)}$

---

13. If $f(x) = x^2 + 2x$, then $\frac{d}{dx}\left(f(\ln x)\right) =$

(A) $\frac{2 \ln x + 2}{x}$ (B) $2x \ln x + 2x$ (C) $2 \ln x + 2$ (D) $2 \ln x + \frac{2}{x}$ (E) $\frac{2x + 2}{x}$

x	0	1	2	3
$f''(x)$	5	0	–7	4

14. The polynomial function f has selected values of its second derivative f'' given in the table above. Which of the following statements must be true?

(A) f is increasing on the interval $(0, 2)$.

(B) f is decreasing on the interval $(0, 2)$.

(C) f has a local maximum at $x = 1$.

(D) The graph of f has a point of inflection at $x = 1$.

(E) The graph of f changes concavity in the interval $(0, 2)$.

---

15. $\displaystyle\int \frac{x}{x^2 - 4}\,dx =$

(A) $\dfrac{-1}{4\left(x^2 - 4\right)^2} + C$

(B) $\dfrac{1}{2\left(x^2 - 4\right)} + C$

(C) $\dfrac{1}{2}\ln\left|x^2 - 4\right| + C$

(D) $2\ln\left|x^2 - 4\right| + C$

(E) $\dfrac{1}{2}\arctan\left(\dfrac{x}{2}\right) + C$

16. If $\sin(xy) = x$, then $\dfrac{dy}{dx} =$

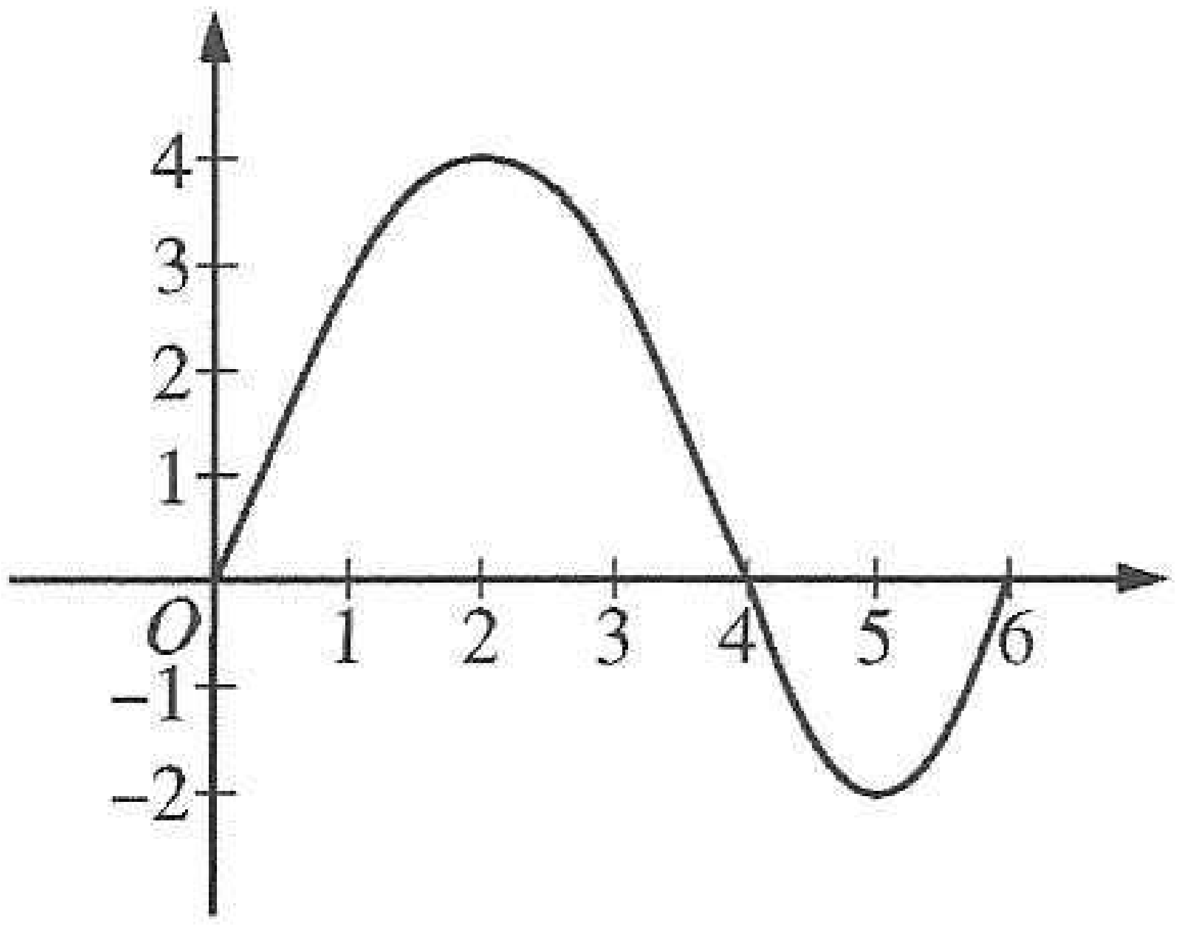
(A) $\dfrac{1}{\cos(xy)}$

(B) $\dfrac{1}{x\cos(xy)}$

(C) $\dfrac{1-\cos(xy)}{\cos(xy)}$

(D) $\dfrac{1-y\cos(xy)}{x\cos(xy)}$

(E) $\dfrac{y(1-\cos(xy))}{x}$



Graph of f

17. The graph of the function f shown above has horizontal tangents at $x = 2$ and $x = 5$. Let g be the function defined by $g(x) = \int_0^x f(t)\,dt$. For what values of x does the graph of g have a point of inflection?

(A) 2 only (B) 4 only (C) 2 and 5 only (D) 2, 4, and 5 (E) 0, 4, and 6

18. In the xy-plane, the line $x + y = k$, where k is a constant, is tangent to the graph of $y = x^2 + 3x + 1$. What is the value of k ?

(A) -3 (B) -2 (C) -1 (D) 0 (E) 1

---

19. What are all horizontal asymptotes of the graph of $y = \dfrac{5 + 2^x}{1 - 2^x}$ in the xy-plane?

(A) $y = -1$ only

(B) $y = 0$ only

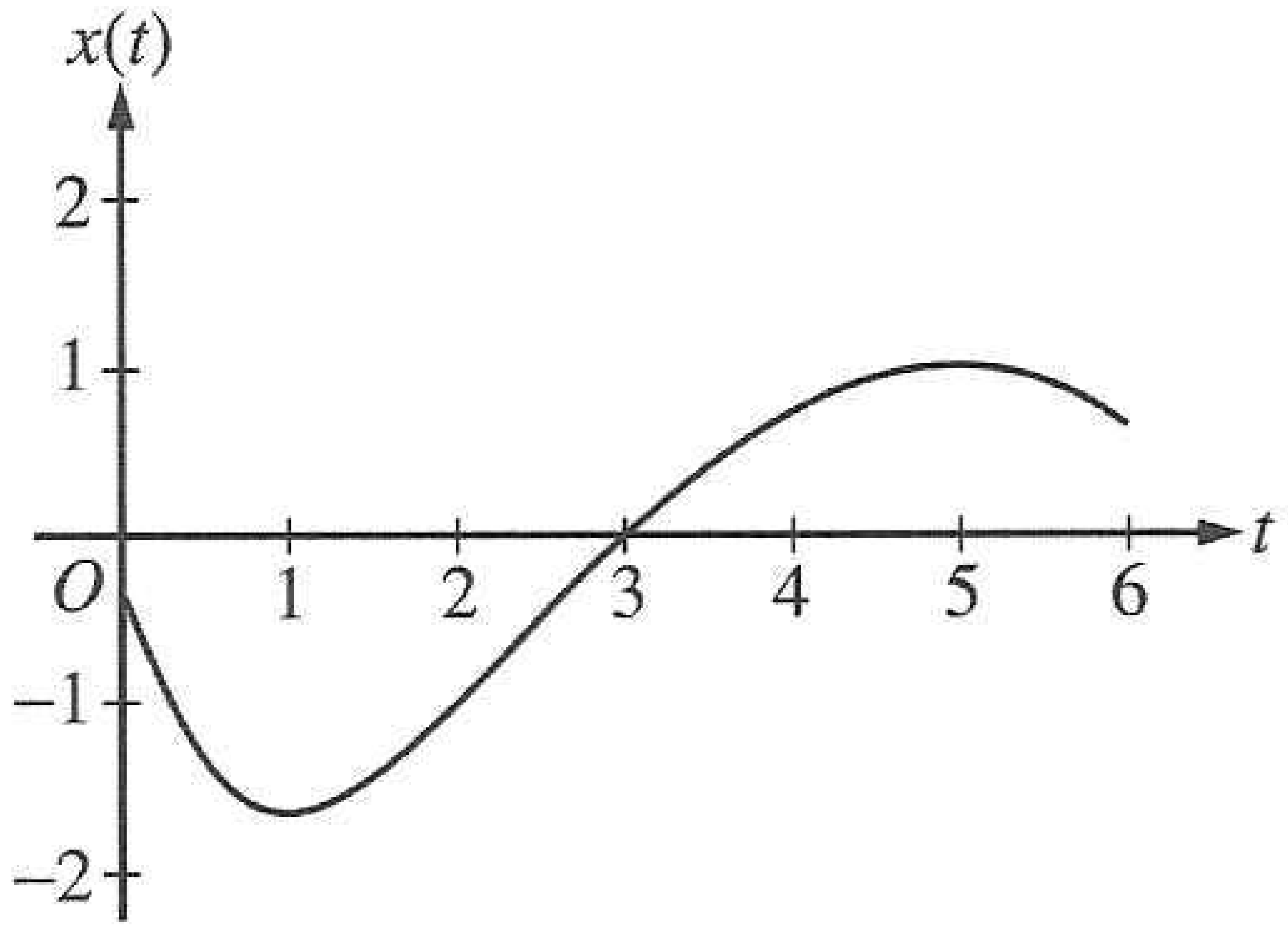
(C) $y = 5$ only

(D) $y = -1$ and $y = 0$

(E) $y = -1$ and $y = 5$

20. Let f be a function with a second derivative given by $f''(x) = x^2(x-3)(x-6)$. What are the x-coordinates of the points of inflection of the graph of f?

(A) 0 only (B) 3 only (C) 0 and 6 only (D) 3 and 6 only (E) 0, 3, and 6



21. A particle moves along a straight line. The graph of the particle's position $x(t)$ at time t is shown above for $0 < t < 6$. The graph has horizontal tangents at $t = 1$ and $t = 5$ and a point of inflection at $t = 2$. For what values of t is the velocity of the particle increasing?

(A) $0 < t < 2$

(B) $1 < t < 5$

(C) $2 < t < 6$

(D) $3 < t < 5$ only

(E) $1 < t < 2$ and $5 < t < 6$

22. A rumor spreads among a population of N people at a rate proportional to the product of the number of people who have heard the rumor and the number of people who have not heard the rumor. If p denotes the number of people who have heard the rumor, which of the following differential equations could be used to model this situation with respect to time t, where k is a positive constant?

(A) $\frac{dp}{dt} = kp$

(B) $\frac{dp}{dt} = kp(N - p)$

(C) $\frac{dp}{dt} = kp(p - N)$

(D) $\frac{dp}{dt} = kt(N - t)$

(E) $\frac{dp}{dt} = kt(t - N)$

23. Which of the following is the solution to the differential equation $\frac{dy}{dx} = \frac{x^2}{y}$ with the initial condition $y(3) = -2$?

(A) $y = 2e^{-9+x^3/3}$

(B) $y = -2e^{-9+x^3/3}$

(C) $y = \sqrt{\frac{2x^3}{3}}$

(D) $y = \sqrt{\frac{2x^3}{3} - 14}$

(E) $y = -\sqrt{\frac{2x^3}{3} - 14}$

---

24. The function f is twice differentiable with $f(2) = 1$, $f'(2) = 4$, and $f''(2) = 3$. What is the value of the approximation of $f(1.9)$ using the line tangent to the graph of f at $x = 2$?

(A) 0.4 (B) 0.6 (C) 0.7 (D) 1.3 (E) 1.4

$$f(x) = \begin{cases} cx + d & \text{for } x \leq 2 \\ x^2 - cx & \text{for } x > 2 \end{cases}$$

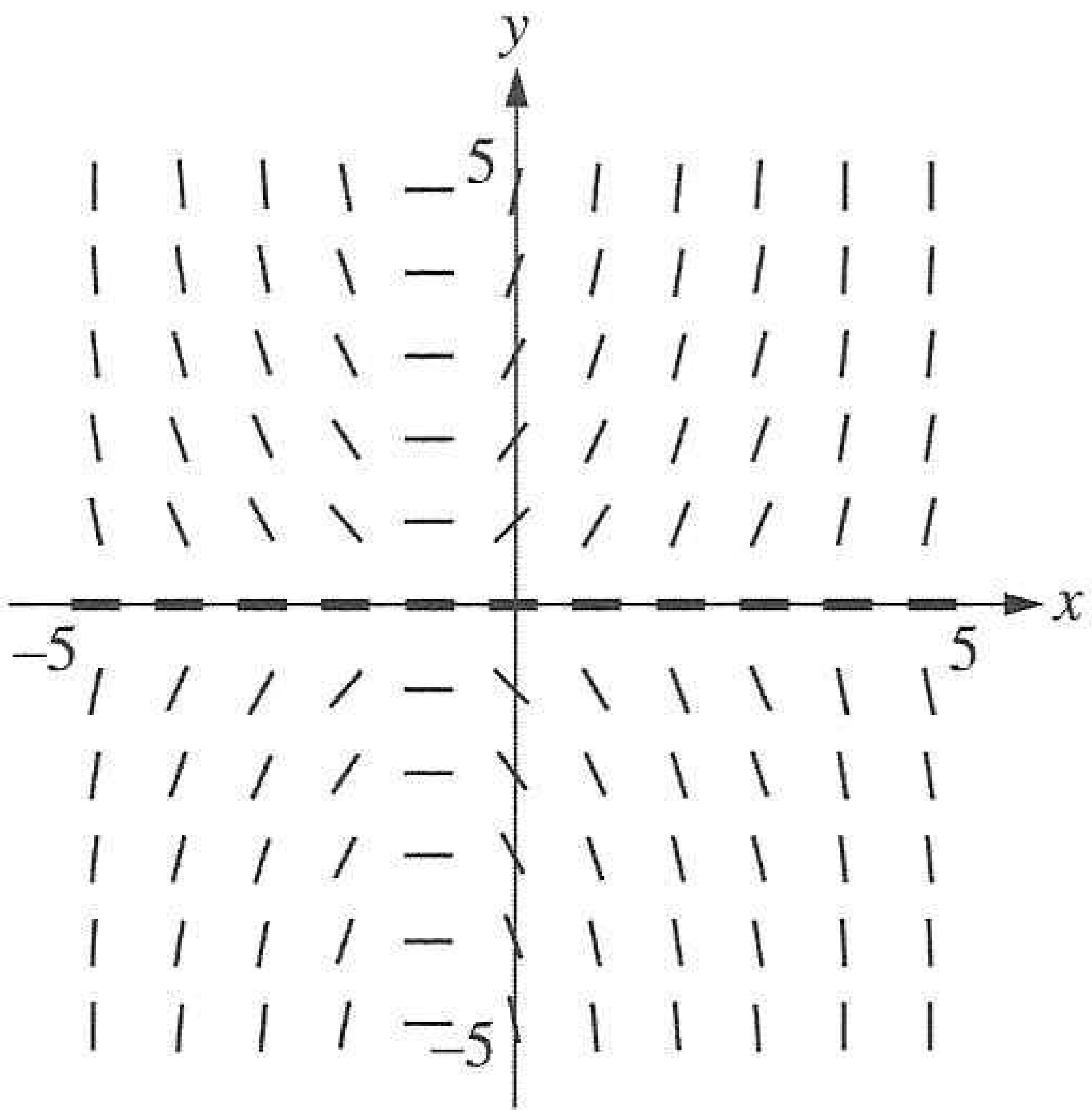
25. Let f be the function defined above, where c and d are constants. If f is differentiable at $x = 2$, what is the value of $c + d$?

(A) -4 (B) -2 (C) 0 (D) 2 (E) 4

---

26. What is the slope of the line tangent to the curve $y = \arctan(4x)$ at the point at which $x = \frac{1}{4}$?

(A) 2 (B) $\frac{1}{2}$ (C) 0 (D) $-\frac{1}{2}$ (E) -2



27. Shown above is a slope field for which of the following differential equations?

(A) $\dfrac{dy}{dx} = xy$

(B) $\dfrac{dy}{dx} = xy - y$

(C) $\dfrac{dy}{dx} = xy + y$

(D) $\dfrac{dy}{dx} = xy + x$

(E) $\dfrac{dy}{dx} = (x + 1)^3$

28. Let f be a differentiable function such that $f(3) = 15$, $f(6) = 3$, $f'(3) = -8$, and $f'(6) = -2$. The function g is differentiable and $g(x) = f^{-1}(x)$ for all x. What is the value of $g'(3)$?

(A) $-\frac{1}{2}$

(B) $-\frac{1}{8}$

(C) $\frac{1}{6}$

(D) $\frac{1}{3}$

(E) The value of $g'(3)$ cannot be determined from the information given.

END OF PART A OF SECTION I

CALCULUS AB
SECTION I, Part B
Time—50 minutes
Number of questions—17

A GRAPHING CALCULATOR IS REQUIRED FOR SOME QUESTIONS ON THIS PART OF THE EXAM.

Directions: Solve each of the following problems, using the available space for scratch work. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding oval on the answer sheet. No credit will be given for anything written in the exam book. Do not spend too much time on any one problem.

BE SURE YOU ARE USING PAGE 3 OF THE ANSWER SHEET TO RECORD YOUR ANSWERS TO QUESTIONS NUMBERED 76-92.

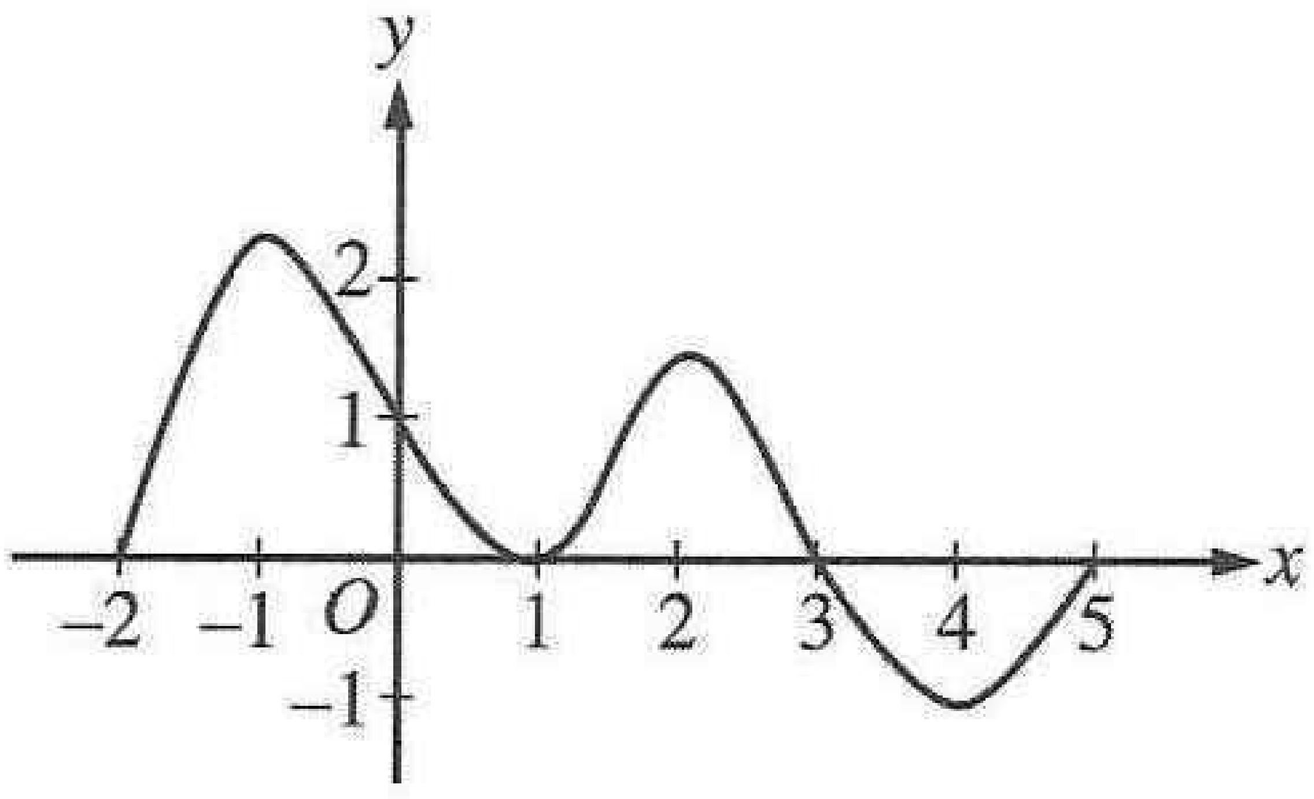
YOU MAY NOT RETURN TO PAGE 2 OF THE ANSWER SHEET.

In this exam:

(1) The exact numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.

(2) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

(3) The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1} x = \arcsin x$).



Graph of f'

76. The graph of f', the derivative of f, is shown above for $-2 \le x \le 5$. On what intervals is f increasing?

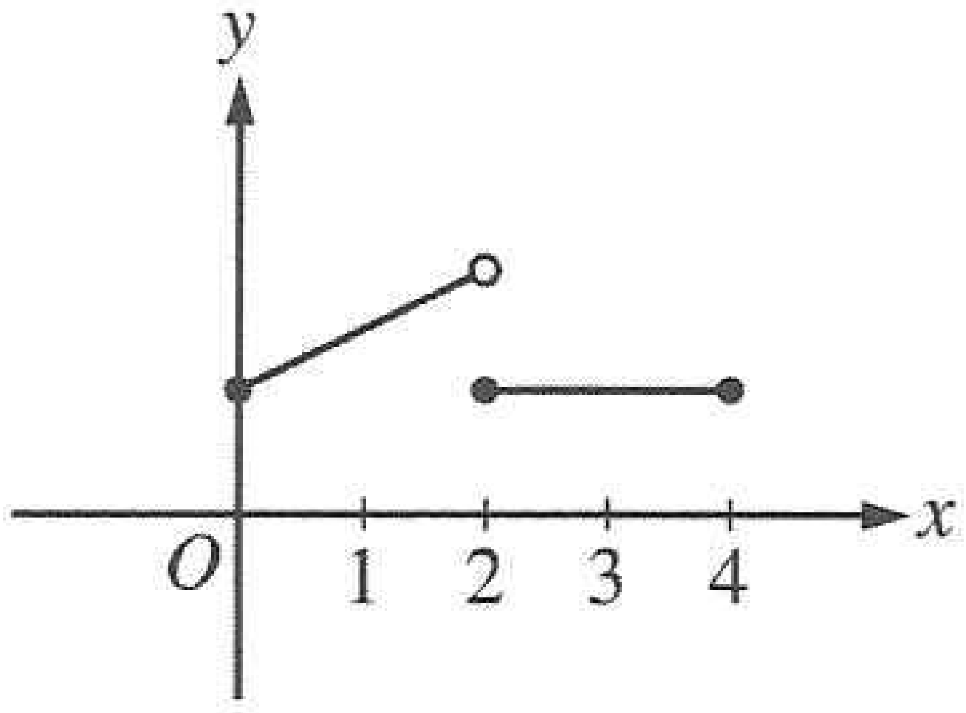
(A) $[-2, 1]$ only

(B) $[-2, 3]$

(C) $[3, 5]$ only

(D) $[0, 1.5]$ and $[3, 5]$

(E) $[-2, -1]$, $[1, 2]$, and $[4, 5]$



Graph of f

77. The figure above shows the graph of a function f with domain $0 \le x \le 4$. Which of the following statements are true?

I. $\lim_{x \to 2^-} f(x)$ exists.

II. $\lim_{x \to 2^+} f(x)$ exists.

III. $\lim_{x \to 2} f(x)$ exists.

(A) I only (B) II only (C) I and II only (D) I and III only (E) I, II, and III

---

78. The first derivative of the function f is defined by $f'(x) = \sin\left(x^3 - x\right)$ for $0 \le x \le 2$. On what intervals is f increasing?

(A) $1 \le x \le 1.445$ only

(B) $1 \le x \le 1.691$

(C) $1.445 \le x \le 1.875$

(D) $0.577 \le x \le 1.445$ and $1.875 \le x \le 2$

(E) $0 \le x \le 1$ and $1.691 \le x \le 2$

79. If $\int_{-5}^{2} f(x)\,dx = -17$ and $\int_{5}^{2} f(x)\,dx = -4$, what is the value of $\int_{-5}^{5} f(x)\,dx$?

(A) -21 (B) -13 (C) 0 (D) 13 (E) 21

---

80. The derivative of the function f is given by $f'(x) = x^2 \cos\left(x^2\right)$. How many points of inflection does the graph of f have on the open interval $(-2, 2)$?

(A) One (B) Two (C) Three (D) Four (E) Five

81. If $G(x)$ is an antiderivative for $f(x)$ and $G(2) = -7$, then $G(4) =$

(A) $f'(4)$

(B) $-7 + f'(4)$

(C) $\int_2^4 f(t)\,dt$

(D) $\int_2^4 (-7 + f(t))\,dt$

(E) $-7 + \int_2^4 f(t)\,dt$

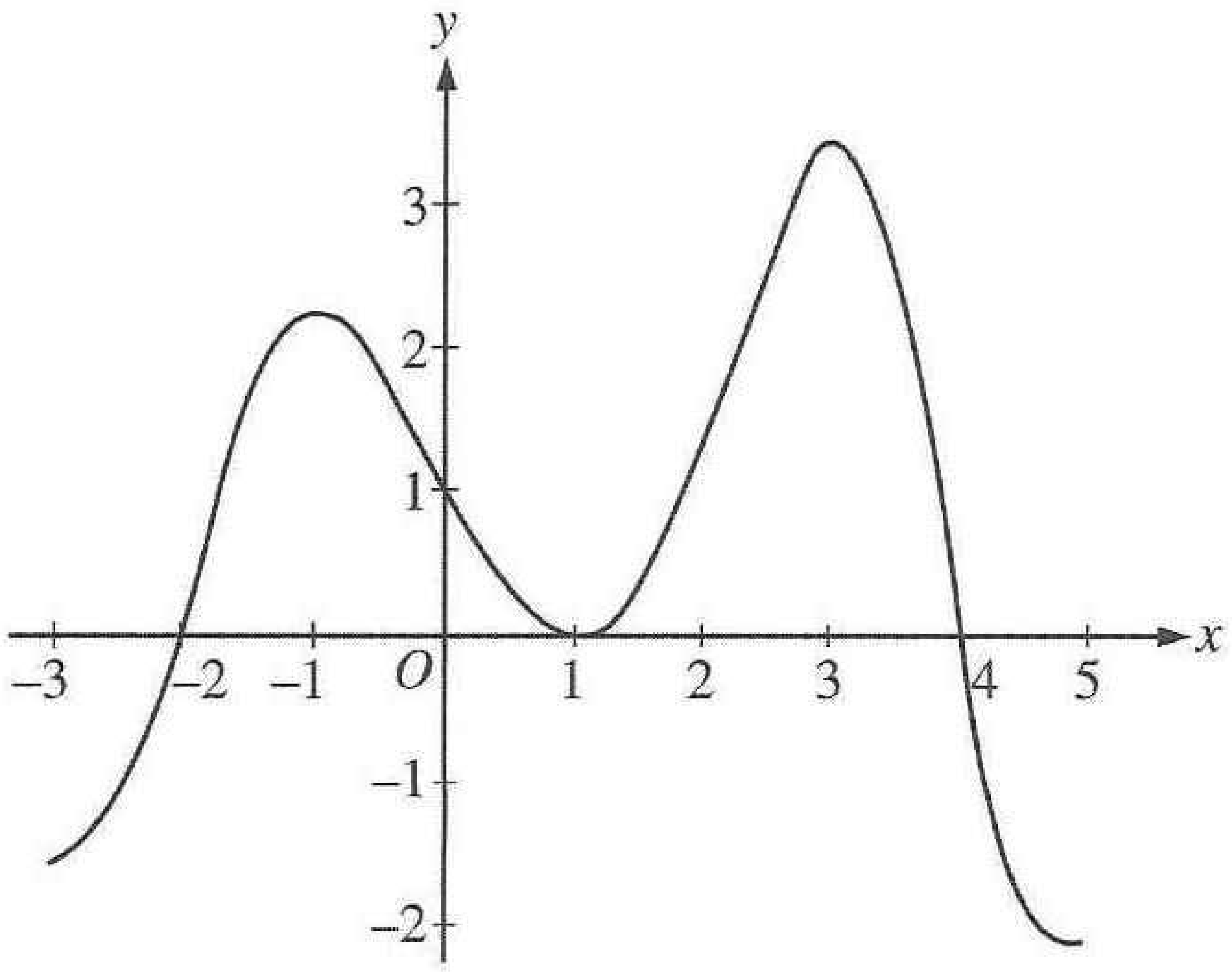
---

82. A particle moves along a straight line with velocity given by $v(t) = 7 - (1.01)^{-t^2}$ at time $t \geq 0$. What is the acceleration of the particle at time $t = 3$?

(A) -0.914 (B) 0.055 (C) 5.486 (D) 6.086 (E) 18.087

83. What is the area enclosed by the curves $y = x^3 - 8x^2 + 18x - 5$ and $y = x + 5$?

(A) 10.667 (B) 11.833 (C) 14.583 (D) 21.333 (E) 32



Graph of f'

84. The graph of the derivative of a function f is shown in the figure above. The graph has horizontal tangent lines at $x = -1$, $x = 1$, and $x = 3$. At which of the following values of x does f have a relative maximum?

(A) −2 only (B) 1 only (C) 4 only (D) −1 and 3 only (E) −2, 1, and 4

x	-4	-3	-2	-1
$f(x)$	0.75	-1.5	-2.25	-1.5
$f'(x)$	-3	-1.5	0	1.5

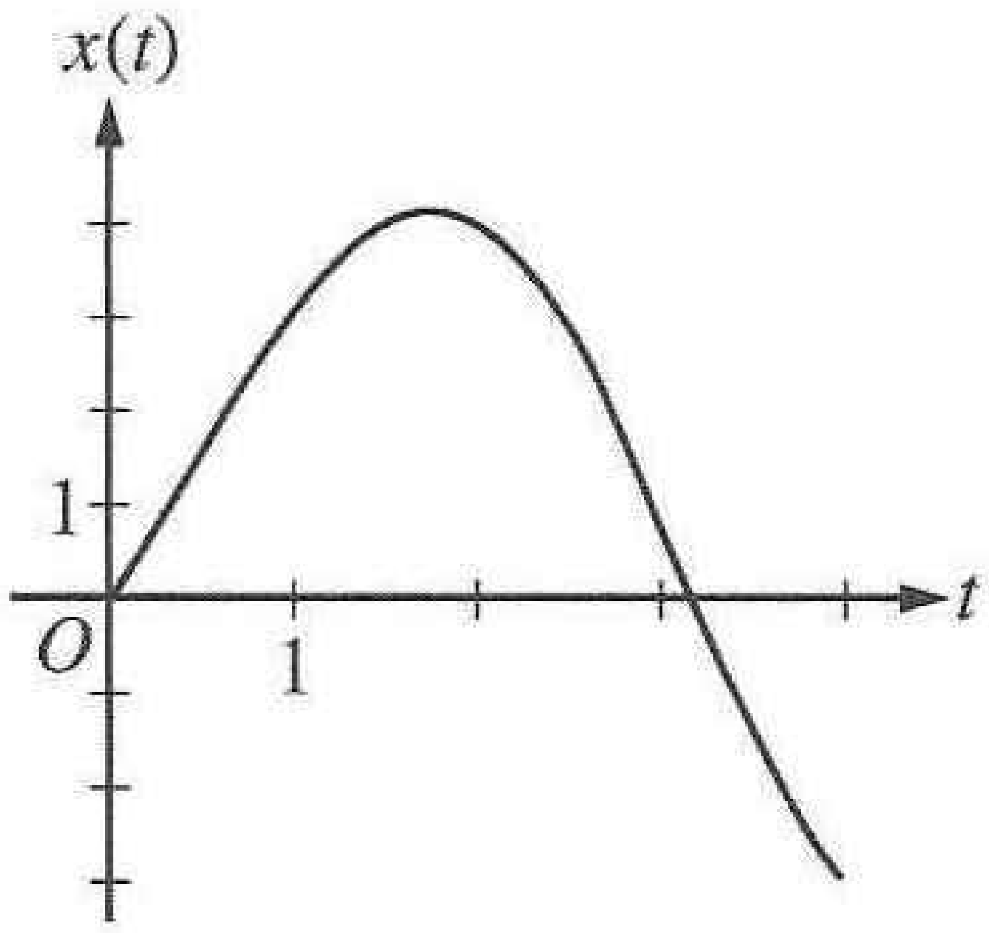
85. The table above gives values of a function f and its derivative at selected values of x. If f' is continuous on the interval $[-4, -1]$, what is the value of $\int_{-4}^{-1} f'(x)\,dx$?

(A) -4.5 (B) -2.25 (C) 0 (D) 2.25 (E) 4.5

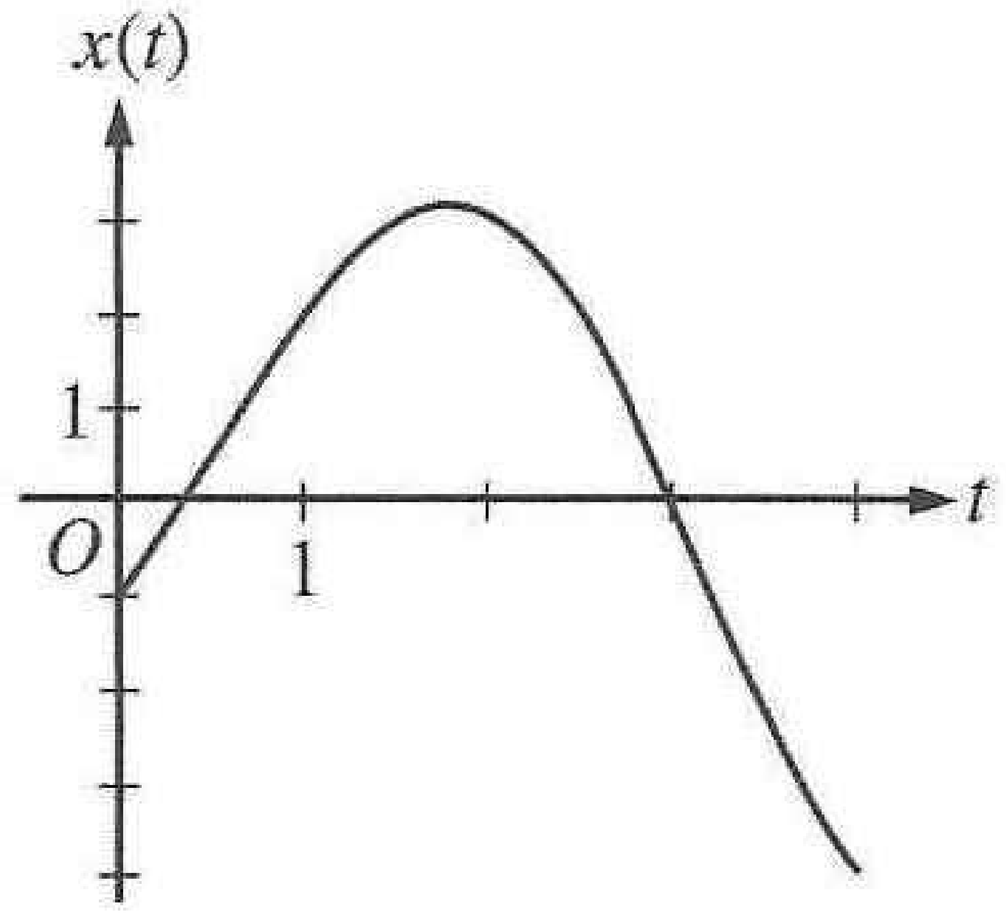
t	0	1	2	3	4
$v(t)$	-1	2	3	0	-4

86. The table gives selected values of the velocity, $v(t)$, of a particle moving along the x-axis. At time $t = 0$, the particle is at the origin. Which of the following could be the graph of the position, $x(t)$, of the particle for $0 \le t \le 4$?

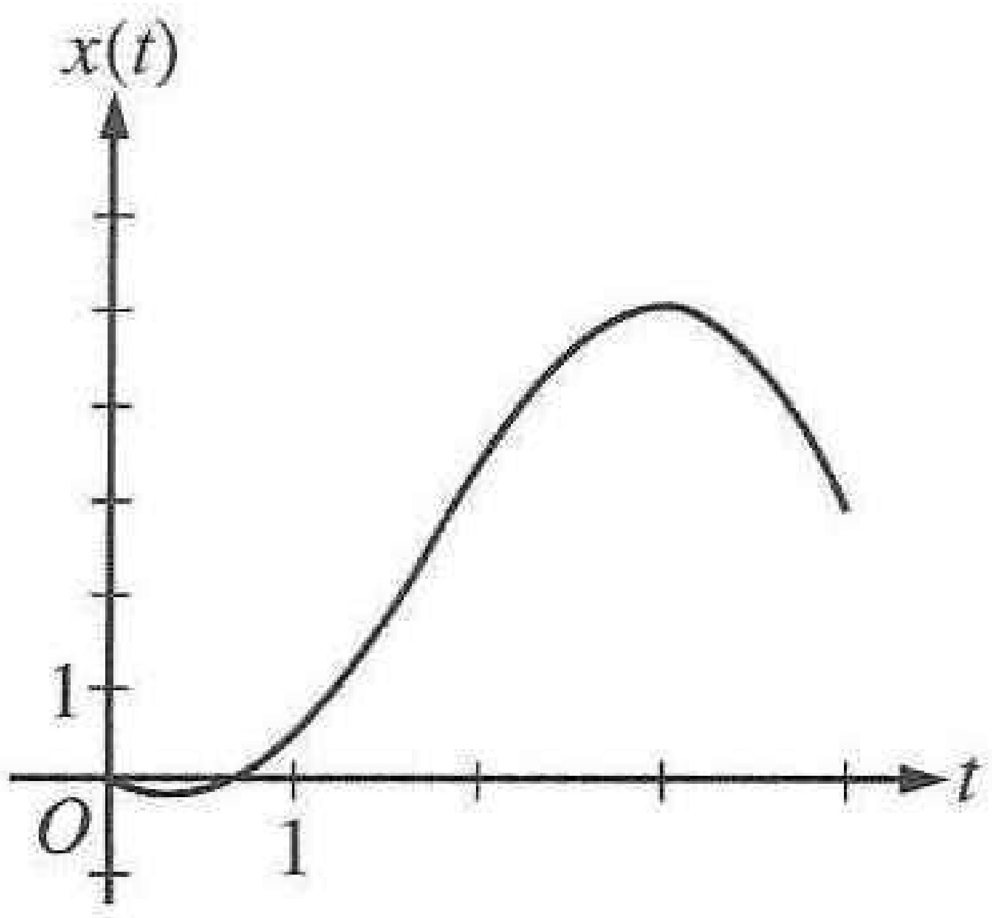
(A)



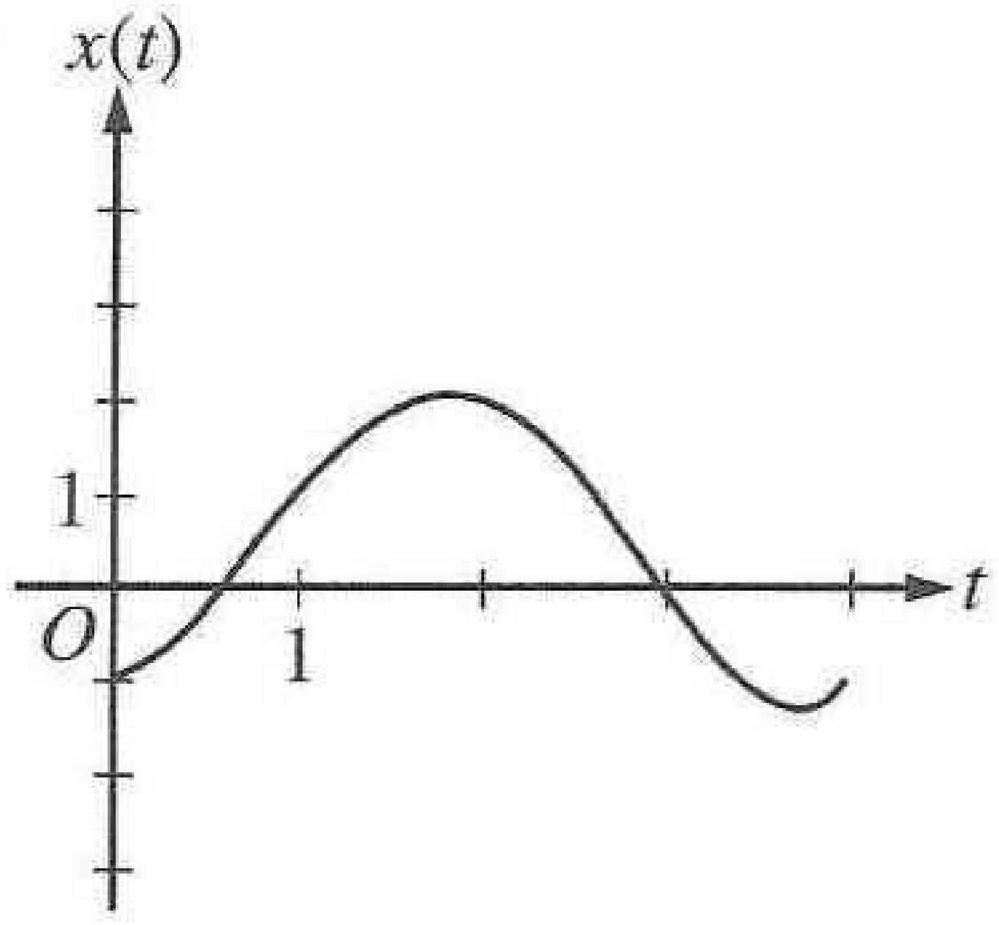
(B)



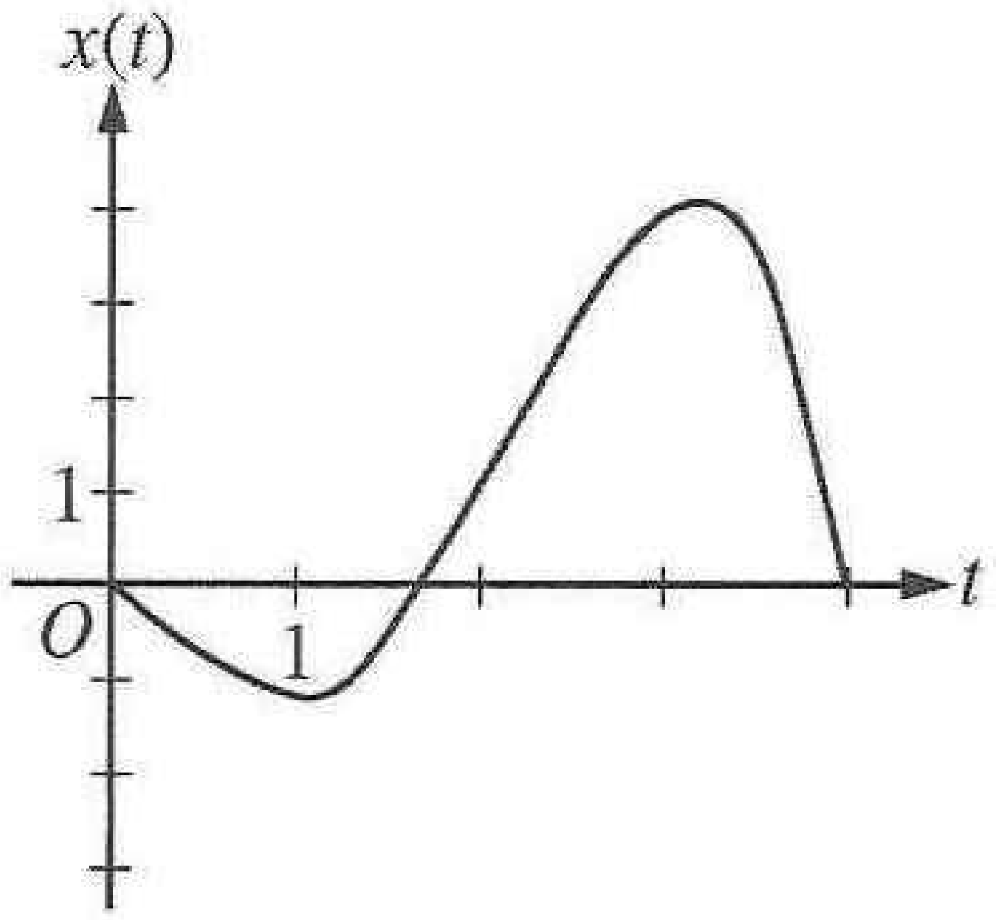
(C)



(D)



(E)



GO ON TO THE NEXT PAGE.

87. An object traveling in a straight line has position $x(t)$ at time t. If the initial position is $x(0) = 2$ and the velocity of the object is $v(t) = \sqrt[3]{1 + t^2}$, what is the position of the object at time $t = 3$?

(A) 0.431 (B) 2.154 (C) 4.512 (D) 6.512 (E) 17.408

---

88. The radius of a sphere is decreasing at a rate of 2 centimeters per second. At the instant when the radius of the sphere is 3 centimeters, what is the rate of change, in square centimeters per second, of the surface area of the sphere? (The surface area S of a sphere with radius r is $S = 4\pi r^2$.)

(A) -108π (B) -72π (C) -48π (D) -24π (E) -16π

89. The function f is continuous for $-2 \le x \le 2$ and $f(-2) = f(2) = 0$. If there is no c, where $-2 < c < 2$, for which $f'(c) = 0$, which of the following statements must be true?

(A) For $-2 < k < 2$, $f'(k) > 0$.

(B) For $-2 < k < 2$, $f'(k) < 0$.

(C) For $-2 < k < 2$, $f'(k)$ exists.

(D) For $-2 < k < 2$, $f'(k)$ exists, but f' is not continuous.

(E) For some k, where $-2 < k < 2$, $f'(k)$ does not exist.

---

90. The function f is continuous on the closed interval $[2, 4]$ and twice differentiable on the open interval $(2, 4)$. If $f'(3) = 2$ and $f''(x) < 0$ on the open interval $(2, 4)$, which of the following could be a table of values for f ?

(A)

x	$f(x)$
2	2.5
3	5
4	6.5

(B)

x	$f(x)$
2	2.5
3	5
4	7

(C)

x	$f(x)$
2	3
3	5
4	6.5

(D)

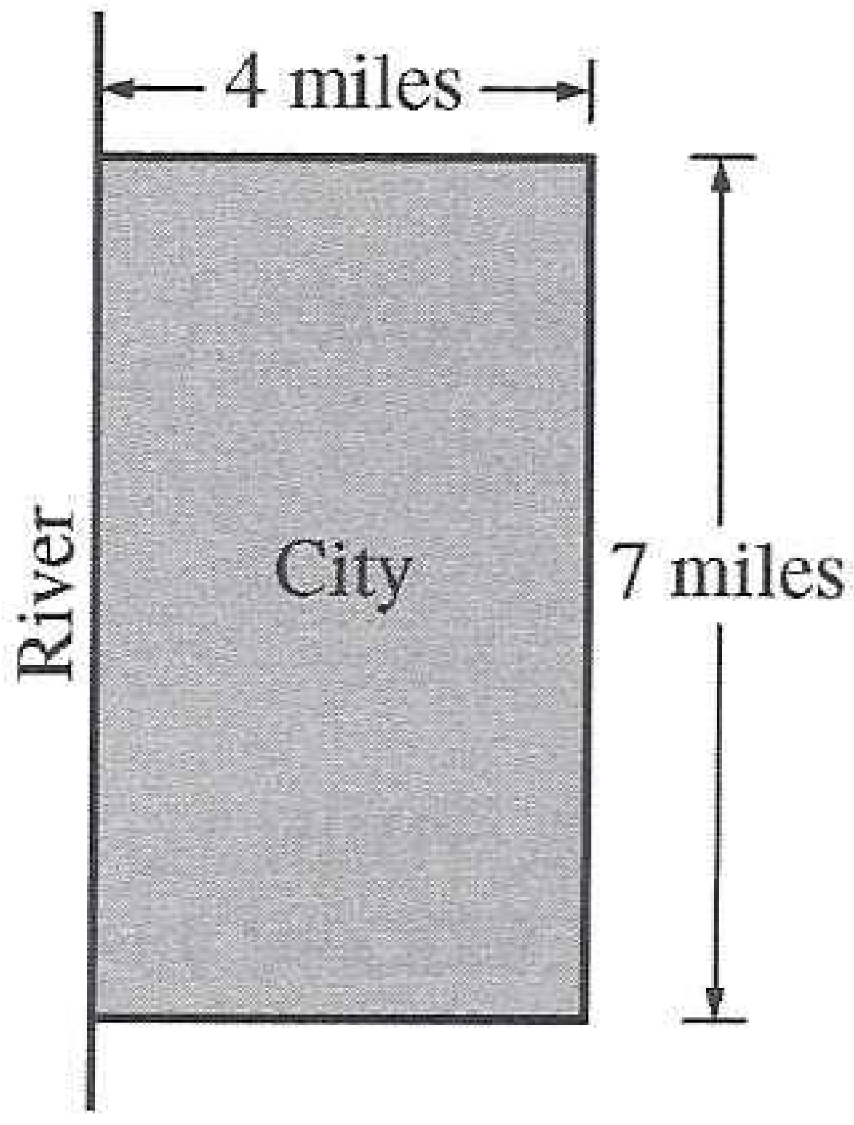
x	$f(x)$
2	3
3	5
4	7

(E)

x	$f(x)$
2	3.5
3	5
4	7.5

91. What is the average value of $y = \dfrac{\cos x}{x^2 + x + 2}$ on the closed interval $[-1, 3]$?

(A) -0.085 (B) 0.090 (C) 0.183 (D) 0.244 (E) 0.732



92. A city located beside a river has a rectangular boundary as shown in the figure above. The population density of the city at any point along a strip x miles from the river's edge is $f(x)$ persons per square mile. Which of the following expressions gives the population of the city?

(A) $\int_0^4 f(x)\,dx$

(B) $7\int_0^4 f(x)\,dx$

(C) $28\int_0^4 f(x)\,dx$

(D) $\int_0^7 f(x)\,dx$

(E) $4\int_0^7 f(x)\,dx$

END OF SECTION I

AFTER TIME HAS BEEN CALLED, TURN TO THE NEXT PAGE AND ANSWER QUESTIONS 93-96.

93. Which graphing calculator did you use during the exam?

(A) Casio 6300, Casio 7300, Casio 7400, Casio 7700, TI-73, TI-80, or TI-81
(B) Casio 9700, Casio 9800, Sharp 9200, Sharp 9300, TI-82, or TI-85
(C) Casio 9750, Casio 9850, Casio 9860, Casio FX 1.0, Sharp 9600, Sharp 9900, TI-83, TI-83 Plus, TI-83 Plus Silver, TI-84 Plus, TI-84 Plus Silver, TI-86, or TI-Nspire
(D) Casio 9970, Casio Algebra FX 2.0, HP 38G, HP 39 series, HP 40G, HP 48 series, HP 49 series, HP 50 series, TI-89, TI-89 Titanium, or TI-Nspire CAS
(E) Some other graphing calculator

94. During your Calculus AB course, which of the following best describes your calculator use?

(A) I used my own graphing calculator.
(B) I used a graphing calculator furnished by my school, both in class and at home.
(C) I used a graphing calculator furnished by my school only in class.
(D) I used a graphing calculator furnished by my school mostly in class, but occasionally at home.
(E) I did not use a graphing calculator.

95. During your Calculus AB course, which of the following describes approximately how often a graphing calculator was used by you or your teacher in classroom learning activities?

(A) Almost every class
(B) About three-quarters of the classes
(C) About one-half of the classes
(D) About one-quarter of the classes
(E) Seldom or never

96. During your Calculus AB course, which of the following describes the portion of testing time you were allowed to use a graphing calculator?

(A) All or almost all of the time
(B) About three-quarters of the time
(C) About one-half of the time
(D) About one-quarter of the time
(E) Seldom or never

AP® Calculus AB Exam

SECTION II: Free-Response Questions

At a Glance

Total Time
1 hour, 30 minutes
Number of Questions
6
Percent of Total Grade
50%
Writing Instrument
Either pencil or pen with black or dark blue ink
Weight
The questions are weighted equally, but the parts of a question are not necessarily given equal weight.

Part A

Number of Questions
3
Time
45 minutes
Electronic Device
Graphing calculator required
Percent of Section II Score
50%

Part B

Number of Questions
3
Time
45 minutes
Electronic Device
None allowed
Percent of Section II Score
50%

Instructions

The questions for Part A are printed in the green insert and the questions for Part B are printed in the blue insert. You may use the inserts to organize your answers and for scratch work, but you must write your answers in the pink Section II booklet. No credit will be given for work written in the inserts. Write your solution to each part of each question in the space provided for that part in the Section II booklet. Write clearly and legibly. Cross out any errors you make; erased or crossed-out work will not be graded.

Manage your time carefully. During the timed portion for Part A, work only on the questions in Part A. You are permitted to use your calculator to solve an equation, find the derivative of a function at a point, or calculate the value of a definite integral. However, you must clearly indicate the setup of your question, namely the equation, function, or integral you are using. If you use other built-in features or programs, you must show the mathematical steps necessary to produce your results. During the timed portion for Part B, you may keep the green insert and continue to work on the questions in Part A without the use of a calculator.

For each part of Section II, you may wish to look over the questions before starting to work on them. It is not expected that everyone will be able to complete all parts of all questions.

- Show all of your work. Clearly label any functions, graphs, tables, or other objects that you use. Your work will be graded on the correctness and completeness of your methods as well as your answers. Answers without supporting work may not receive credit. Justifications require that you give mathematical (noncalculator) reasons.
- Your work must be expressed in standard mathematical notation rather than calculator syntax. For example, $\int_1^5 x^2\,dx$ may not be written as fnInt(X^2, X, 1, 5).
- Unless otherwise specified, answers (numeric or algebraic) need not be simplified. If you use decimal approximations in calculations, your work will be graded on accuracy. Unless otherwise specified, your final answers should be accurate to three places after the decimal point.
- Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

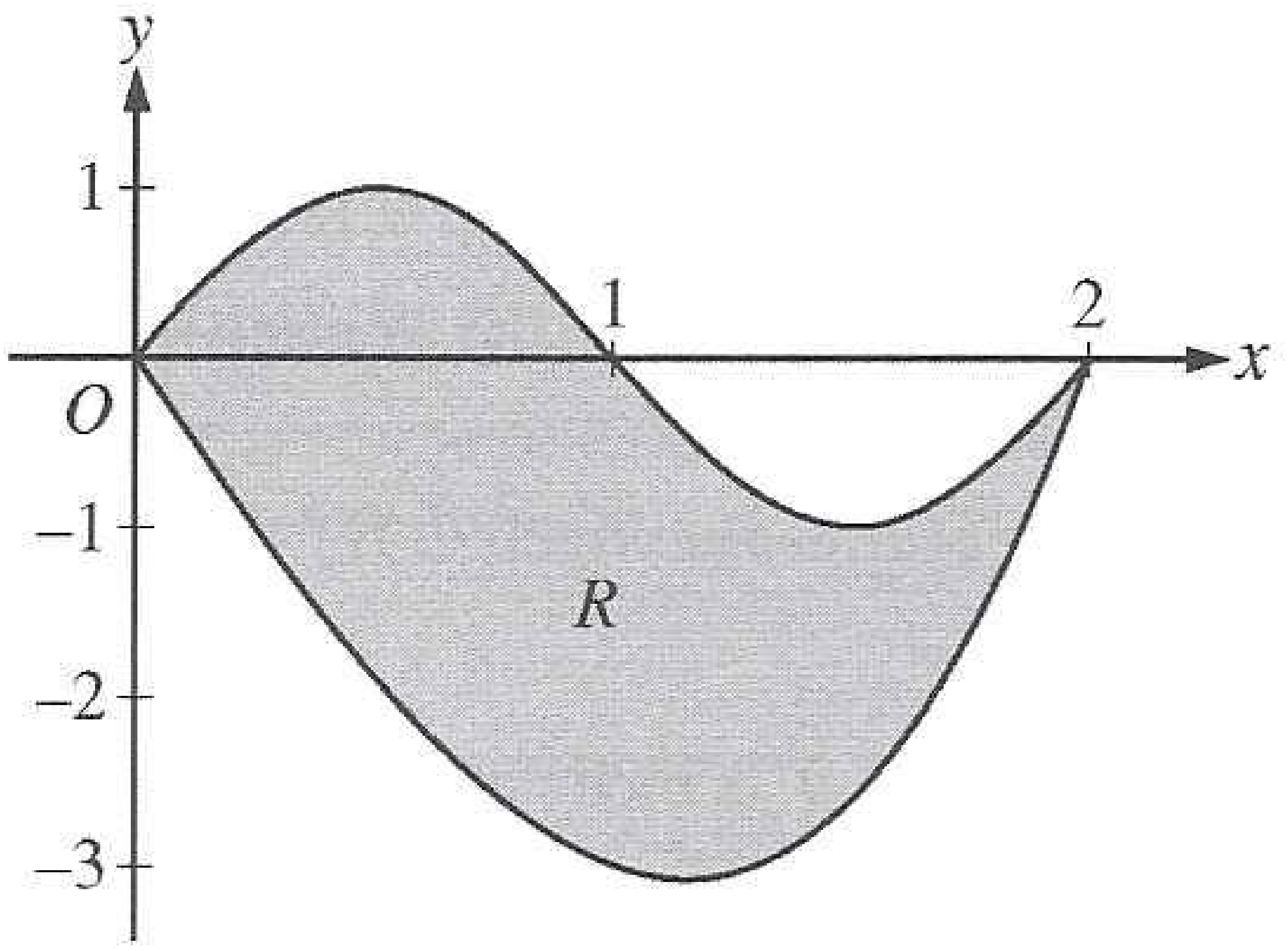
CALCULUS AB

SECTION II, Part A

Time—45 minutes

Number of problems—3

A graphing calculator is required for some problems or parts of problems.



1. Let R be the region bounded by the graphs of $y = \sin(\pi x)$ and $y = x^3 - 4x$, as shown in the figure above.

   (a) Find the area of R.

   (b) The horizontal line $y = -2$ splits the region R into two parts. Write, but do not evaluate, an integral expression for the area of the part of R that is below this horizontal line.

   (c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a square. Find the volume of this solid.

   (d) The region R models the surface of a small pond. At all points in R at a distance x from the y-axis, the depth of the water is given by $h(x) = 3 - x$. Find the volume of water in the pond.

t (hours)	0	1	3	4	7	8	9
$L(t)$ (people)	120	156	176	126	150	80	0

2. Concert tickets went on sale at noon $(t = 0)$ and were sold out within 9 hours. The number of people waiting in line to purchase tickets at time t is modeled by a twice-differentiable function L for $0 \le t \le 9$. Values of $L(t)$ at various times t are shown in the table above.

   (a) Use the data in the table to estimate the rate at which the number of people waiting in line was changing at 5:30 P.M. $(t = 5.5)$. Show the computations that lead to your answer. Indicate units of measure.

   (b) Use a trapezoidal sum with three subintervals to estimate the average number of people waiting in line during the first 4 hours that tickets were on sale.

   (c) For $0 \le t \le 9$, what is the fewest number of times at which $L'(t)$ must equal 0 ? Give a reason for your answer.

   (d) The rate at which tickets were sold for $0 \le t \le 9$ is modeled by $r(t) = 550te^{-t/2}$ tickets per hour. Based on the model, how many tickets were sold by 3 P.M. $(t = 3)$, to the nearest whole number?

3. Oil is leaking from a pipeline on the surface of a lake and forms an oil slick whose volume increases at a constant rate of 2000 cubic centimeters per minute. The oil slick takes the form of a right circular cylinder with both its radius and height changing with time. (Note: The volume V of a right circular cylinder with radius r and height h is given by $V = \pi r^2 h$.)

   (a) At the instant when the radius of the oil slick is 100 centimeters and the height is 0.5 centimeter, the radius is increasing at the rate of 2.5 centimeters per minute. At this instant, what is the rate of change of the height of the oil slick with respect to time, in centimeters per minute?

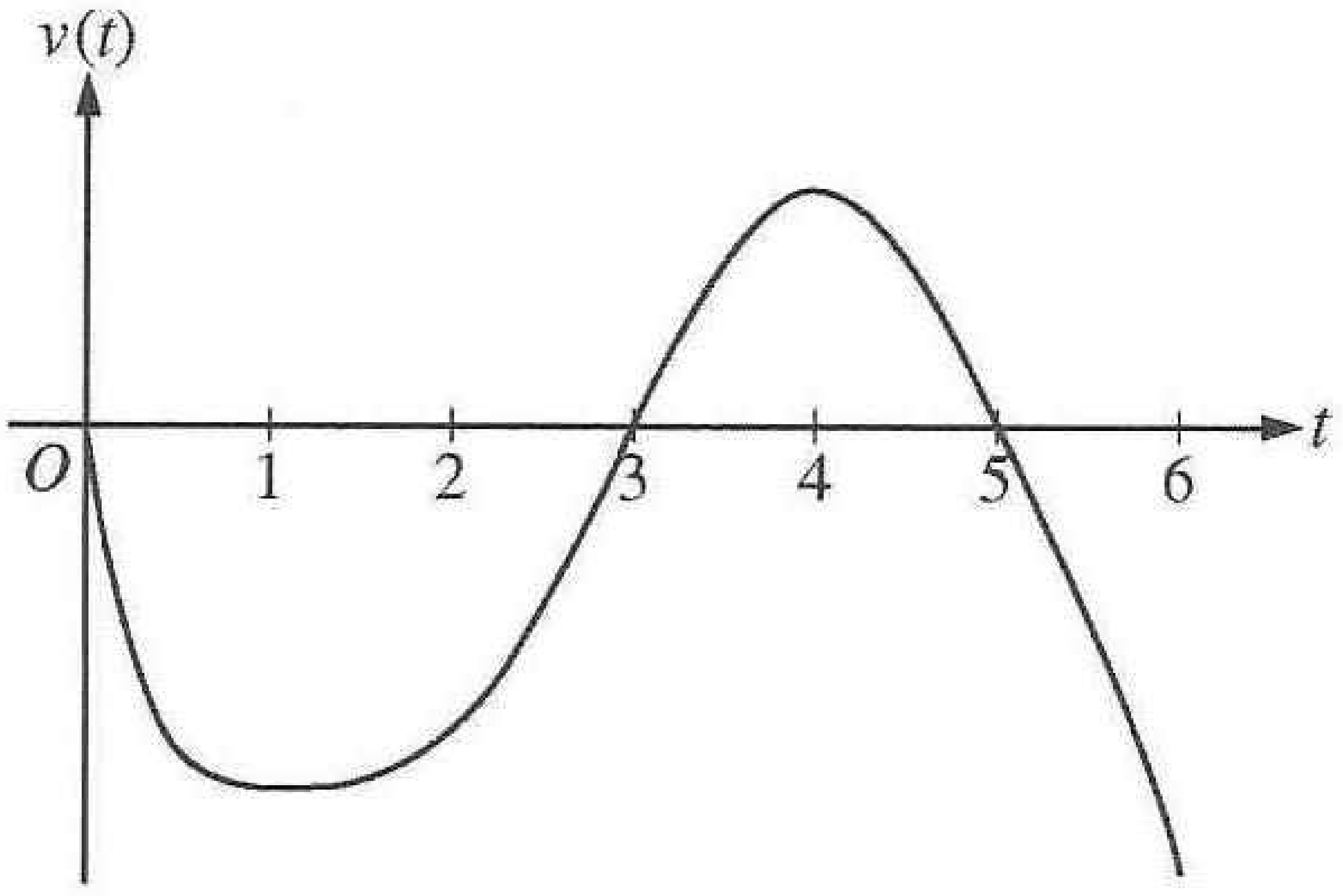
   (b) A recovery device arrives on the scene and begins removing oil. The rate at which oil is removed is $R(t) = 400\sqrt{t}$ cubic centimeters per minute, where t is the time in minutes since the device began working. Oil continues to leak at the rate of 2000 cubic centimeters per minute. Find the time t when the oil slick reaches its maximum volume. Justify your answer.

   (c) By the time the recovery device began removing oil, 60,000 cubic centimeters of oil had already leaked. Write, but do not evaluate, an expression involving an integral that gives the volume of oil at the time found in part (b).

END OF PART A OF SECTION II

CALCULUS AB
SECTION II, Part B
Time—45 minutes
Number of problems—3

No calculator is allowed for these problems.



Graph of v

4. A particle moves along the x-axis so that its velocity at time t, for $0 \le t \le 6$, is given by a differentiable function v whose graph is shown above. The velocity is 0 at $t = 0$, $t = 3$, and $t = 5$, and the graph has horizontal tangents at $t = 1$ and $t = 4$. The areas of the regions bounded by the t-axis and the graph of v on the intervals $[0, 3]$, $[3, 5]$, and $[5, 6]$ are 8, 3, and 2, respectively. At time $t = 0$, the particle is at $x = -2$.

   (a) For $0 \le t \le 6$, find both the time and the position of the particle when the particle is farthest to the left. Justify your answer.

   (b) For how many values of t, where $0 \le t \le 6$, is the particle at $x = -8$? Explain your reasoning.

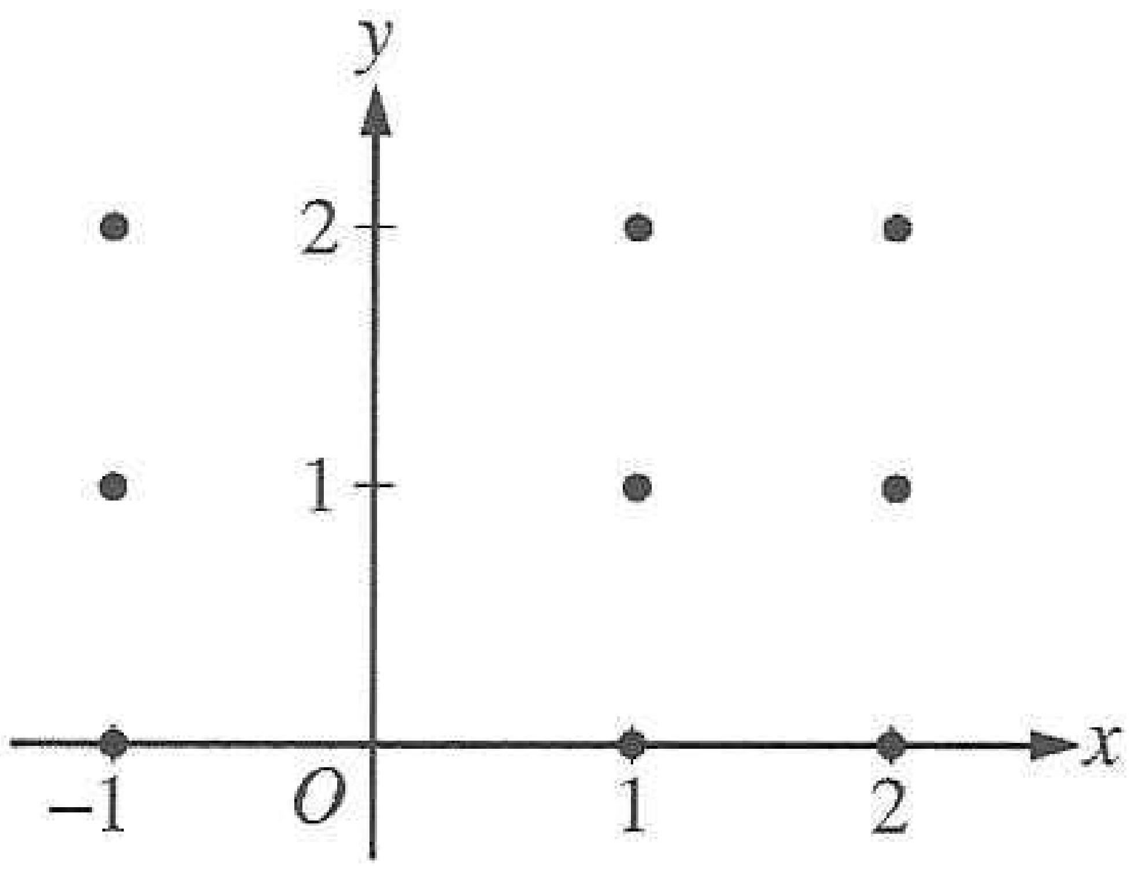
   (c) On the interval $2 < t < 3$, is the speed of the particle increasing or decreasing? Give a reason for your answer.

   (d) During what time intervals, if any, is the acceleration of the particle negative? Justify your answer.

5. Consider the differential equation $\frac{dy}{dx} = \frac{y-1}{x^2}$, where $x \neq 0$.

   (a) On the axes provided, sketch a slope field for the given differential equation at the nine points indicated.

   (Note: Use the axes provided in the exam booklet.)



   (b) Find the particular solution $y = f(x)$ to the differential equation with the initial condition $f(2) = 0$.

   (c) For the particular solution $y = f(x)$ described in part (b), find $\lim_{x \to \infty} f(x)$.

---

6. Let f be the function given by $f(x) = \frac{\ln x}{x}$ for all $x > 0$. The derivative of f is given by $f'(x) = \frac{1 - \ln x}{x^2}$.

   (a) Write an equation for the line tangent to the graph of f at $x = e^2$.

   (b) Find the x-coordinate of the critical point of f. Determine whether this point is a relative minimum, a relative maximum, or neither for the function f. Justify your answer.

   (c) The graph of the function f has exactly one point of inflection. Find the x-coordinate of this point.

   (d) Find $\lim_{x \to 0^+} f(x)$.

END OF EXAM

AP® Calculus BC Exam

SECTION I: Multiple-Choice Questions

At a Glance

Total Time
1 hour, 45 minutes
Number of Questions
45
Percent of Total Grade
50%
Writing Instrument
Pencil required

Part A

Number of Questions
28
Time
55 minutes
Electronic Device
None allowed

Part B

Number of Questions
17
Time
50 minutes
Electronic Device
Graphing calculator required

Instructions

Section I of this exam contains 45 multiple-choice questions and 4 survey questions. For Part A, fill in only the ovals for numbers 1 through 28 on page 2 of the answer sheet. For Part B, fill in only the ovals for numbers 76 through 92 on page 3 of the answer sheet. The survey questions are numbers 93 through 96.

Indicate all of your answers to the multiple-choice questions on the answer sheet. No credit will be given for anything written in this exam booklet, but you may use the booklet for notes or scratch work. After you have decided which of the suggested answers is best, completely fill in the corresponding oval on the answer sheet. Give only one answer to each question. If you change an answer, be sure that the previous mark is erased completely. Here is a sample question and answer.

Sample Question	Sample Answer
Chicago is a (A) state (B) city (C) country (D) continent (E) village	Ⓐ ● Ⓒ Ⓓ Ⓔ

Use your time effectively, working as quickly as you can without losing accuracy. Do not spend too much time on any one question. Go on to other questions and come back to the ones you have not answered if you have time. It is not expected that everyone will know the answers to all of the multiple-choice questions.

About Guessing

Many students wonder whether or not to guess the answers to questions about which they are not certain. In this section of the exam, as a correction for random guessing, one-fourth of the number of questions you answer incorrectly will be subtracted from the number of questions you answer correctly. If you are not sure of the best answer but have some knowledge of the question and are able to eliminate one or more of the answer choices, your chance of answering correctly is improved, and it may be to your advantage to answer such a question.

CALCULUS BC
SECTION I, Part A
Time—55 minutes
Number of questions—28

A CALCULATOR MAY NOT BE USED ON THIS PART OF THE EXAM.

Directions: Solve each of the following problems, using the available space for scratch work. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding oval on the answer sheet. No credit will be given for anything written in the exam book. Do not spend too much time on any one problem.

In this exam:

(1) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

(2) The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1} x = \arcsin x$).

GO ON TO THE NEXT PAGE.

1. At time $t \geq 0$, a particle moving in the xy-plane has velocity vector given by $v(t) = \left\langle t^2, 5t \right\rangle$. What is the acceleration vector of the particle at time $t = 3$?

(A) $\left\langle 9, \frac{45}{2} \right\rangle$ (B) $\langle 6, 5 \rangle$ (C) $\langle 2, 0 \rangle$ (D) $\sqrt{306}$ (E) $\sqrt{61}$

---

2. $\int xe^{x^2}\,dx =$

(A) $\frac{1}{2}e^{x^2} + C$ (B) $e^{x^2} + C$ (C) $xe^{x^2} + C$ (D) $\frac{1}{2}e^{2x} + C$ (E) $e^{2x} + C$

---

3. $\lim_{x \to 0} \frac{\sin x \cos x}{x}$ is

(A) -1 (B) 0 (C) 1 (D) $\frac{\pi}{4}$ (E) nonexistent

GO ON TO THE NEXT PAGE.

4. Consider the series $\sum_{n=1}^{\infty} \frac{e^n}{n!}$. If the ratio test is applied to the series, which of the following inequalities results, implying that the series converges?

(A) $\lim_{n\to\infty} \frac{e}{n!} < 1$

(B) $\lim_{n\to\infty} \frac{n!}{e} < 1$

(C) $\lim_{n\to\infty} \frac{n+1}{e} < 1$

(D) $\lim_{n\to\infty} \frac{e}{n+1} < 1$

(E) $\lim_{n\to\infty} \frac{e}{(n+1)!} < 1$

---

5. Which of the following gives the length of the path described by the parametric equations $x = \sin\left(t^3\right)$ and $y = e^{5t}$ from $t = 0$ to $t = \pi$?

(A) $\int_0^{\pi} \sqrt{\sin^2\left(t^3\right) + e^{10t}}\, dt$

(B) $\int_0^{\pi} \sqrt{\cos^2\left(t^3\right) + e^{10t}}\, dt$

(C) $\int_0^{\pi} \sqrt{9t^4 \cos^2\left(t^3\right) + 25e^{10t}}\, dt$

(D) $\int_0^{\pi} \sqrt{3t^2 \cos\left(t^3\right) + 5e^{5t}}\, dt$

(E) $\int_0^{\pi} \sqrt{\cos^2\left(3t^2\right) + e^{10t}}\, dt$

$$f(x) = \begin{cases} \dfrac{x^2 - 4}{x - 2} & \text{if } x \neq 2 \\ 1 & \text{if } x = 2 \end{cases}$$

6. Let f be the function defined above. Which of the following statements about f are true?

I. f has a limit at $x = 2$.

II. f is continuous at $x = 2$.

III. f is differentiable at $x = 2$.

(A) I only

(B) II only

(C) III only

(D) I and II only

(E) I, II, and III

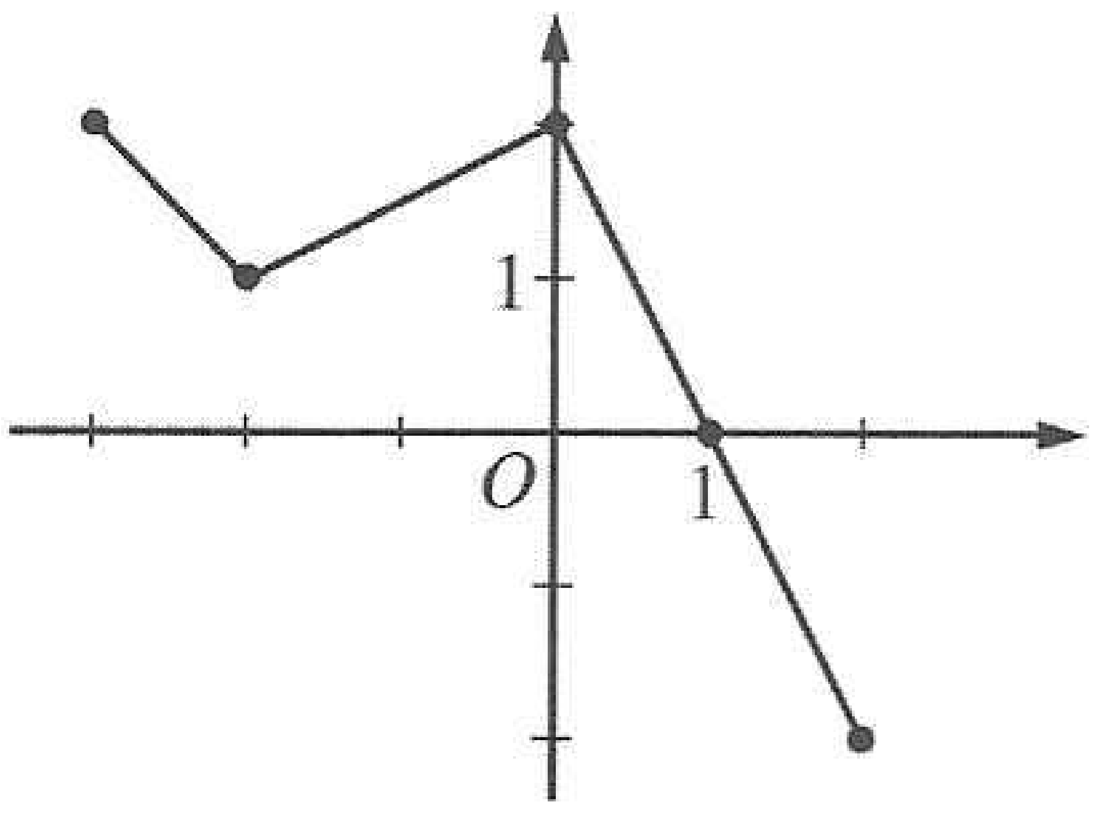
7. Given that $y(1) = -3$ and $\dfrac{dy}{dx} = 2x + y$, what is the approximation for $y(2)$ if Euler's method is used with a step size of 0.5, starting at $x = 1$?

(A) -5 (B) -4.25 (C) -4 (D) -3.75 (E) -3.5

x	2	3	5	8	13
$f(x)$	6	–2	–1	3	9

8. The function f is continuous on the closed interval $[2, 13]$ and has values as shown in the table above. Using the intervals $[2, 3]$, $[3, 5]$, $[5, 8]$, and $[8, 13]$, what is the approximation of $\int_2^{13} f(x)\,dx$ obtained from a left Riemann sum?

(A) 6 (B) 14 (C) 28 (D) 32 (E) 50



Graph of f

9. The graph of the piecewise linear function f is shown in the figure above. If $g(x) = \int_{-2}^{x} f(t)\,dt$, which of the following values is greatest?

(A) $g(-3)$ (B) $g(-2)$ (C) $g(0)$ (D) $g(1)$ (E) $g(2)$

10. In the xy-plane, what is the slope of the line tangent to the graph of $x^2 + xy + y^2 = 7$ at the point $(2, 1)$?

(A) $-\frac{4}{3}$ (B) $-\frac{5}{4}$ (C) -1 (D) $-\frac{4}{5}$ (E) $-\frac{3}{4}$

---

11. Let R be the region between the graph of $y = e^{-2x}$ and the x-axis for $x \geq 3$. The area of R is

(A) $\frac{1}{2e^6}$ (B) $\frac{1}{e^6}$ (C) $\frac{2}{e^6}$ (D) $\frac{\pi}{2e^6}$ (E) infinite

12. Which of the following series converges for all real numbers x ?

(A) $\sum_{n=1}^{\infty} \frac{x^n}{n}$

(B) $\sum_{n=1}^{\infty} \frac{x^n}{n^2}$

(C) $\sum_{n=1}^{\infty} \frac{x^n}{\sqrt{n}}$

(D) $\sum_{n=1}^{\infty} \frac{e^n x^n}{n!}$

(E) $\sum_{n=1}^{\infty} \frac{n! x^n}{e^n}$

---

13. $\int_1^e \frac{x^2+1}{x}\,dx =$

(A) $\frac{e^2-1}{2}$ (B) $\frac{e^2+1}{2}$ (C) $\frac{e^2+2}{2}$ (D) $\frac{e^2-1}{e^2}$ (E) $\frac{2e^2-8e+6}{3e}$

x	0	1	2	3
$f''(x)$	5	0	–7	4

14. The polynomial function f has selected values of its second derivative f'' given in the table above. Which of the following statements must be true?

(A) f is increasing on the interval $(0, 2)$.

(B) f is decreasing on the interval $(0, 2)$.

(C) f has a local maximum at $x = 1$.

(D) The graph of f has a point of inflection at $x = 1$.

(E) The graph of f changes concavity in the interval $(0, 2)$.

---

15. If $f(x) = (\ln x)^2$, then $f''(\sqrt{e}) =$

(A) $\frac{1}{e}$ (B) $\frac{2}{e}$ (C) $\frac{1}{2\sqrt{e}}$ (D) $\frac{1}{\sqrt{e}}$ (E) $\frac{2}{\sqrt{e}}$

GO ON TO THE NEXT PAGE.

16. What are all values of x for which the series $\sum_{n=1}^{\infty}\left(\frac{2}{x^2+1}\right)^n$ converges?

(A) $-1 < x < 1$

(B) $x > 1$ only

(C) $x \geq 1$ only

(D) $x < -1$ and $x > 1$ only

(E) $x \leq -1$ and $x \geq 1$

---

17. Let h be a differentiable function, and let f be the function defined by $f(x) = h\left(x^2 - 3\right)$. Which of the following is equal to $f'(2)$?

(A) $h'(1)$ (B) $4h'(1)$ (C) $4h'(2)$ (D) $h'(4)$ (E) $4h'(4)$

18. In the xy-plane, the line $x + y = k$, where k is a constant, is tangent to the graph of $y = x^2 + 3x + 1$. What is the value of k?

(A) -3 (B) -2 (C) -1 (D) 0 (E) 1

---

19. $$\int \frac{7x}{(2x-3)(x+2)}\,dx =$$

(A) $\frac{3}{2}\ln|2x-3| + 2\ln|x+2| + C$

(B) $3\ln|2x-3| + 2\ln|x+2| + C$

(C) $3\ln|2x-3| - 2\ln|x+2| + C$

(D) $-\frac{6}{(2x-3)^2} - \frac{2}{(x+2)^2} + C$

(E) $-\frac{3}{(2x-3)^2} - \frac{2}{(x+2)^2} + C$

GO ON TO THE NEXT PAGE.

20. What is the sum of the series $1 + \ln 2 + \frac{(\ln 2)^2}{2!} + \cdots + \frac{(\ln 2)^n}{n!} + \cdots$?

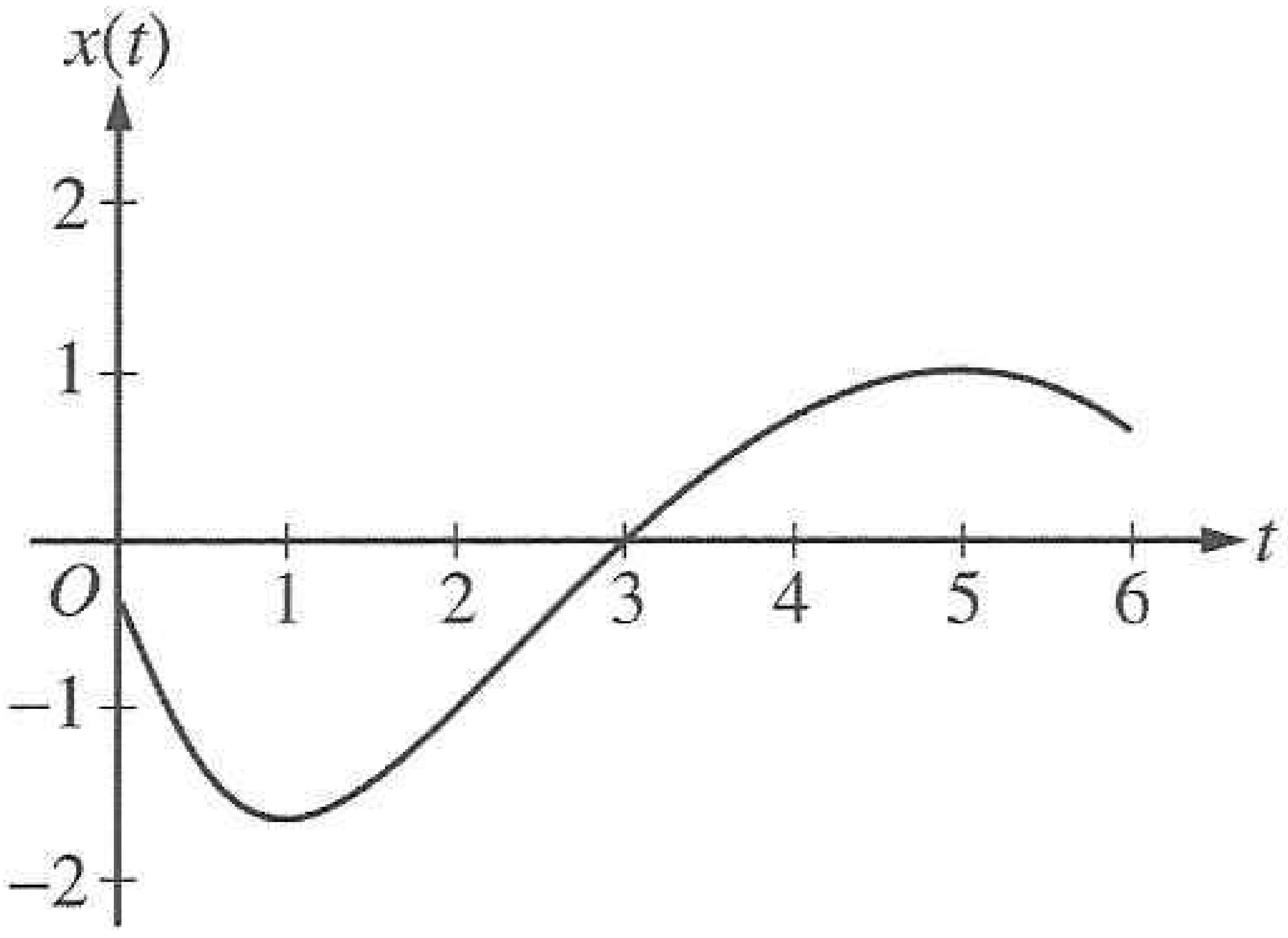
(A) $\ln 2$

(B) $\ln(1 + \ln 2)$

(C) 2

(D) e^2

(E) The series diverges.



21. A particle moves along a straight line. The graph of the particle's position $x(t)$ at time t is shown above for $0 < t < 6$. The graph has horizontal tangents at $t = 1$ and $t = 5$ and a point of inflection at $t = 2$. For what values of t is the velocity of the particle increasing?

(A) $0 < t < 2$

(B) $1 < t < 5$

(C) $2 < t < 6$

(D) $3 < t < 5$ only

(E) $1 < t < 2$ and $5 < t < 6$

x	0	1
$f(x)$	2	4
$f'(x)$	6	–3
$g(x)$	–4	3
$g'(x)$	2	–1

22. The table above gives values of f, f', g, and g' for selected values of x. If $\int_0^1 f'(x)g(x)\,dx = 5$, then $\int_0^1 f(x)g'(x)\,dx =$

(A) –14 (B) –13 (C) –2 (D) 7 (E) 15

---

23. If $f(x) = x\sin(2x)$, which of the following is the Taylor series for f about $x = 0$?

(A) $x - \frac{x^3}{2!} + \frac{x^5}{4!} - \frac{x^7}{6!} + \cdots$

(B) $x - \frac{4x^3}{2!} + \frac{16x^5}{4!} - \frac{64x^7}{6!} + \cdots$

(C) $2x - \frac{8x^3}{3!} + \frac{32x^5}{5!} - \frac{128x^7}{7!} + \cdots$

(D) $2x^2 - \frac{2x^4}{3!} + \frac{2x^6}{5!} - \frac{2x^8}{7!} + \cdots$

(E) $2x^2 - \frac{8x^4}{3!} + \frac{32x^6}{5!} - \frac{128x^8}{7!} + \cdots$

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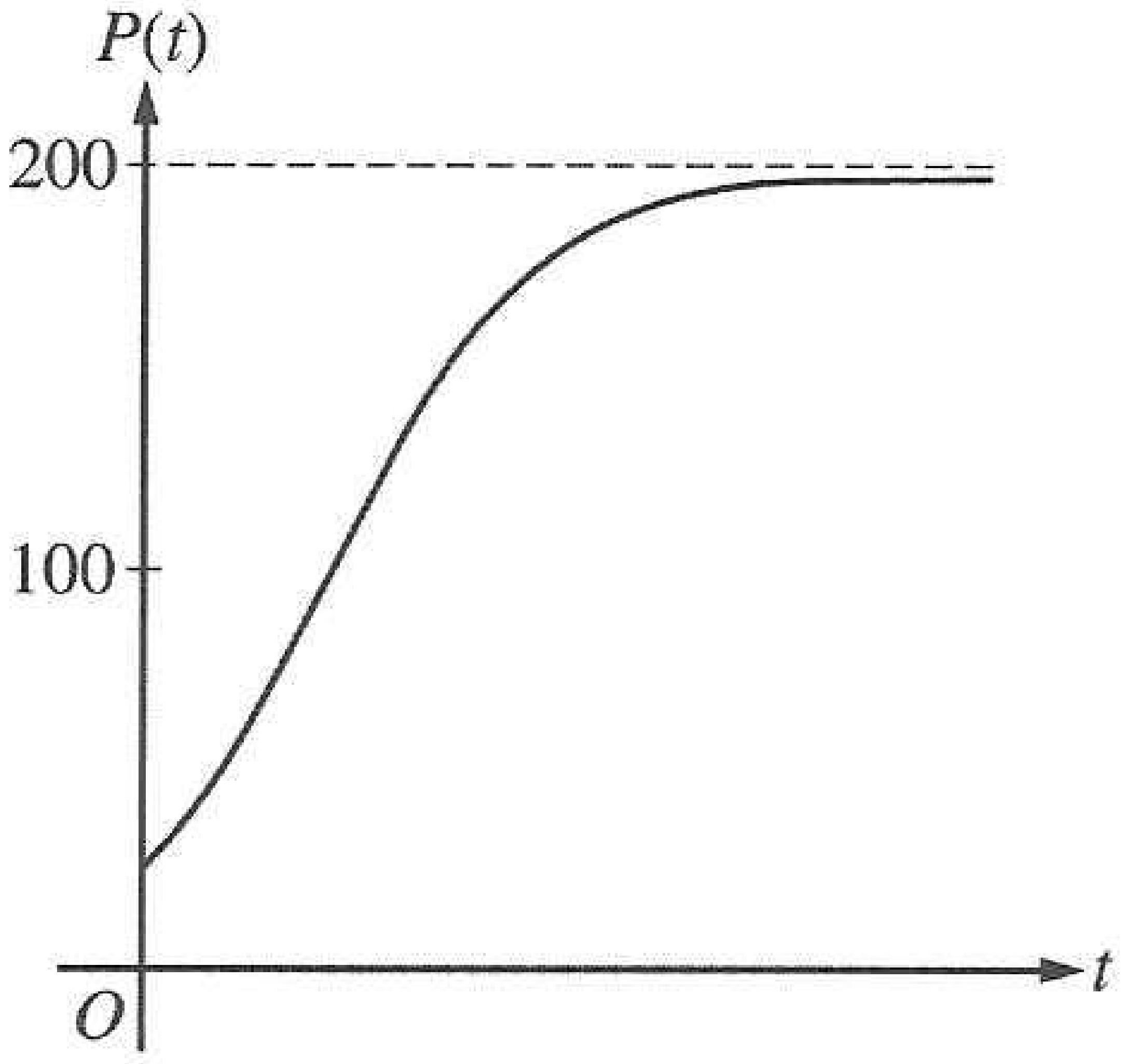


24. Which of the following differential equations for a population P could model the logistic growth shown in the figure above?

(A) $\dfrac{dP}{dt} = 0.2P - 0.001P^2$

(B) $\dfrac{dP}{dt} = 0.1P - 0.001P^2$

(C) $\dfrac{dP}{dt} = 0.2P^2 - 0.001P$

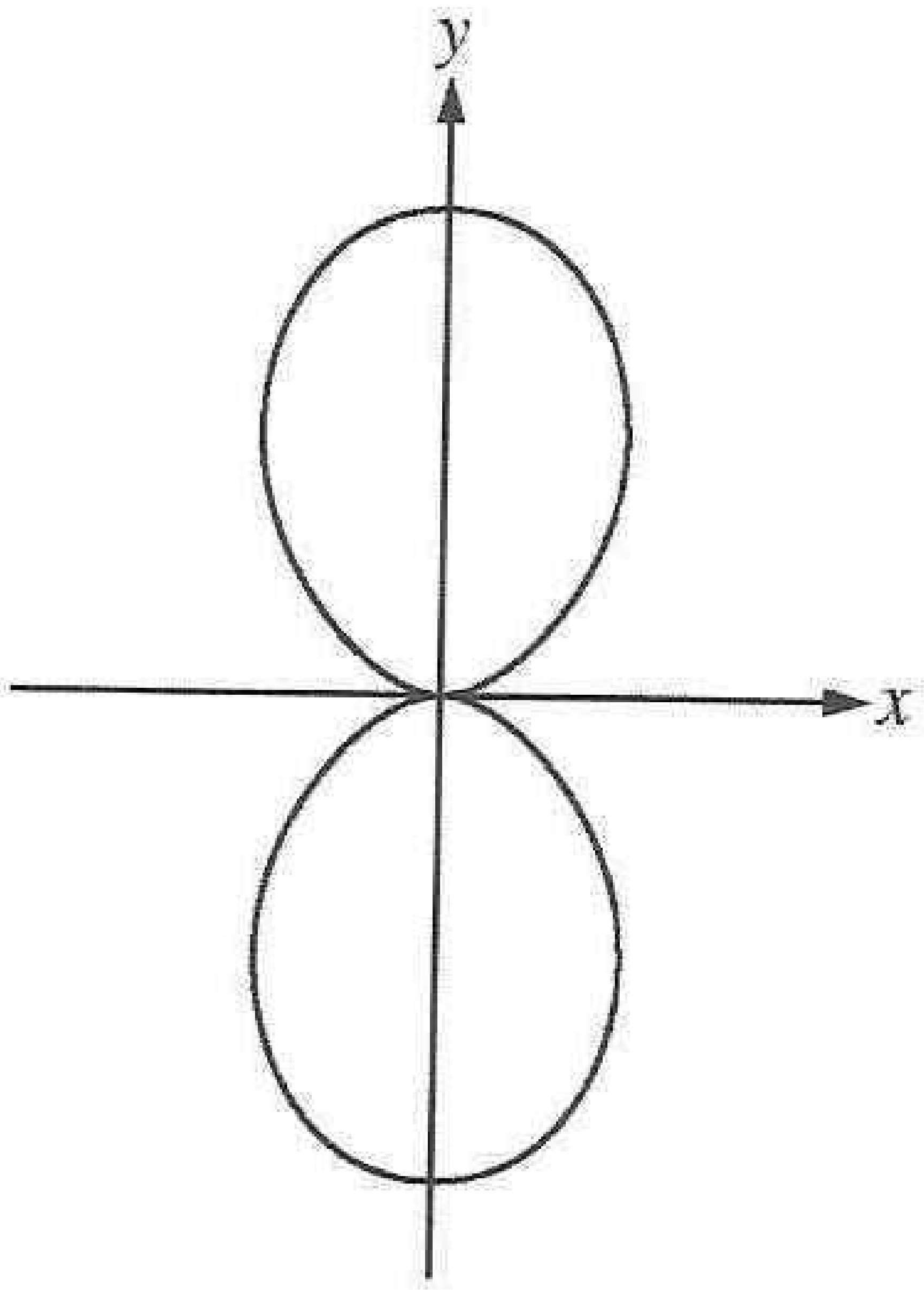
(D) $\dfrac{dP}{dt} = 0.1P^2 - 0.001P$

(E) $\dfrac{dP}{dt} = 0.1P^2 + 0.001P$

$$f(x) = \begin{cases} cx + d & \text{for } x \le 2 \\ x^2 - cx & \text{for } x > 2 \end{cases}$$

25. Let f be the function defined above, where c and d are constants. If f is differentiable at $x = 2$, what is the value of $c + d$?

(A) -4 (B) -2 (C) 0 (D) 2 (E) 4



26. Which of the following expressions gives the total area enclosed by the polar curve $r = \sin^2\theta$ shown in the figure above?

(A) $\frac{1}{2}\int_0^{\pi} \sin^2\theta \, d\theta$

(B) $\int_0^{\pi} \sin^2\theta \, d\theta$

(C) $\frac{1}{2}\int_0^{\pi} \sin^4\theta \, d\theta$

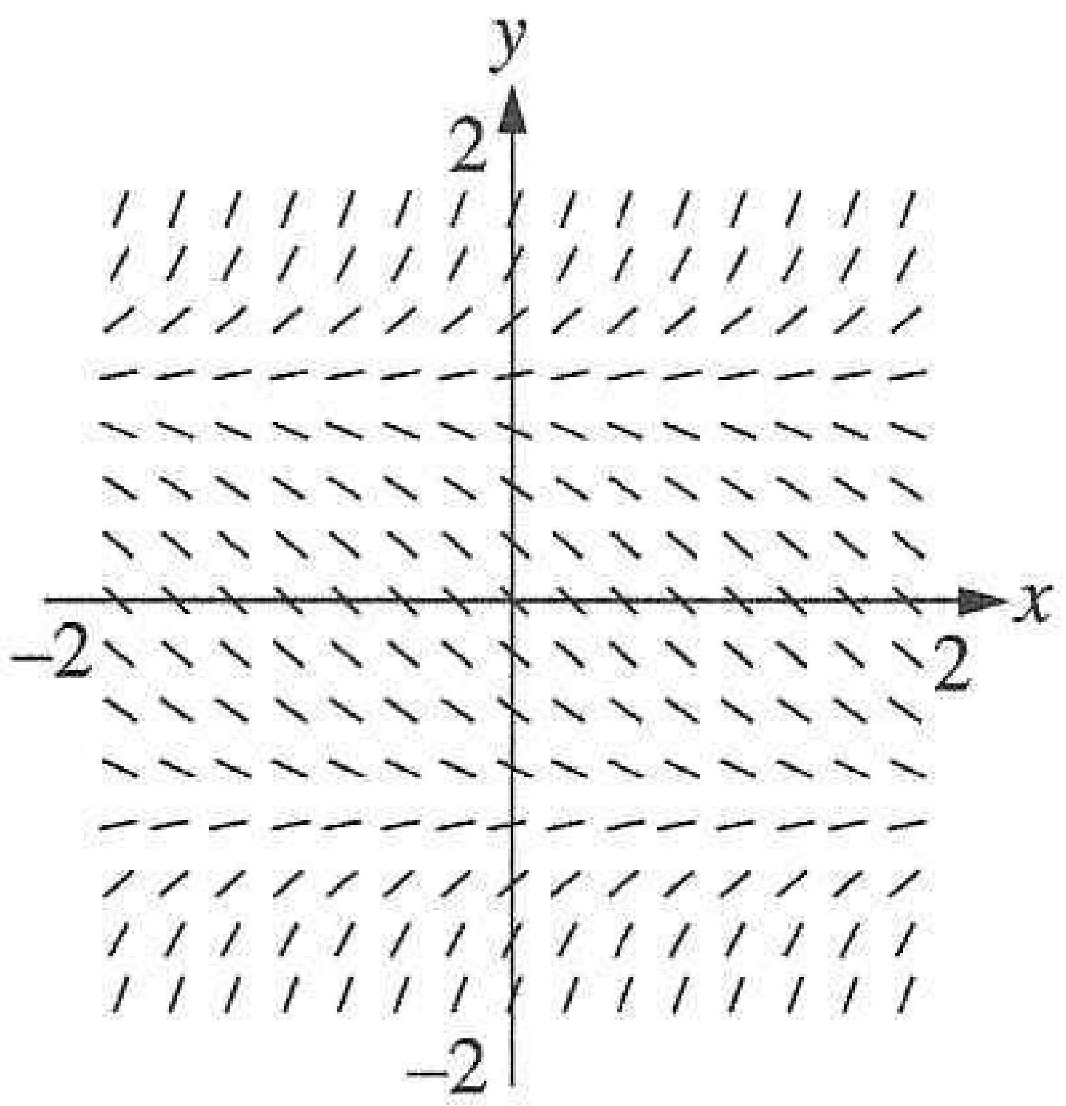
(D) $\int_0^{\pi} \sin^4\theta \, d\theta$

(E) $2\int_0^{\pi} \sin^4\theta \, d\theta$

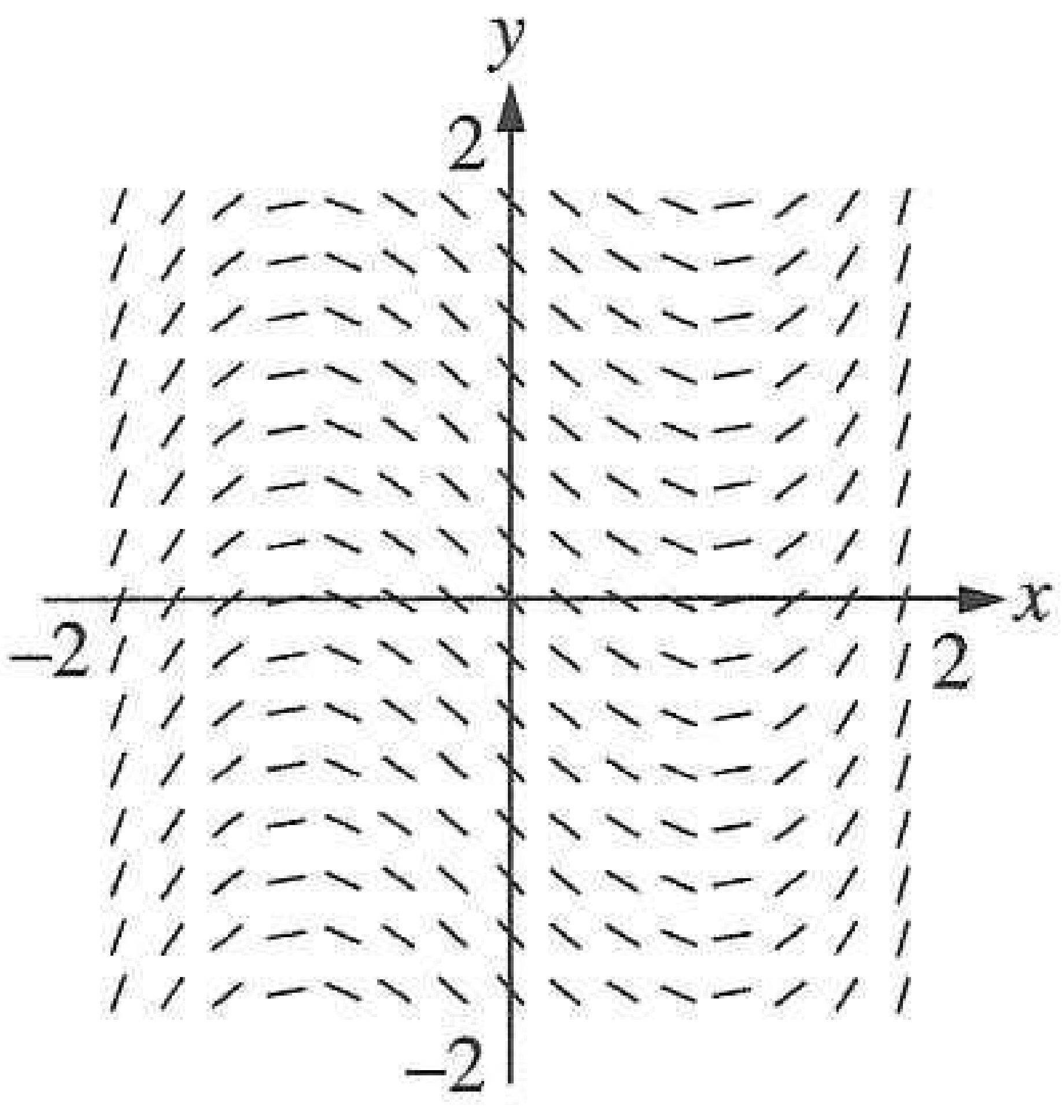
GO ON TO THE NEXT PAGE.

27. Which of the following could be the slope field for the differential equation $\frac{dy}{dx} = y^2 - 1$?

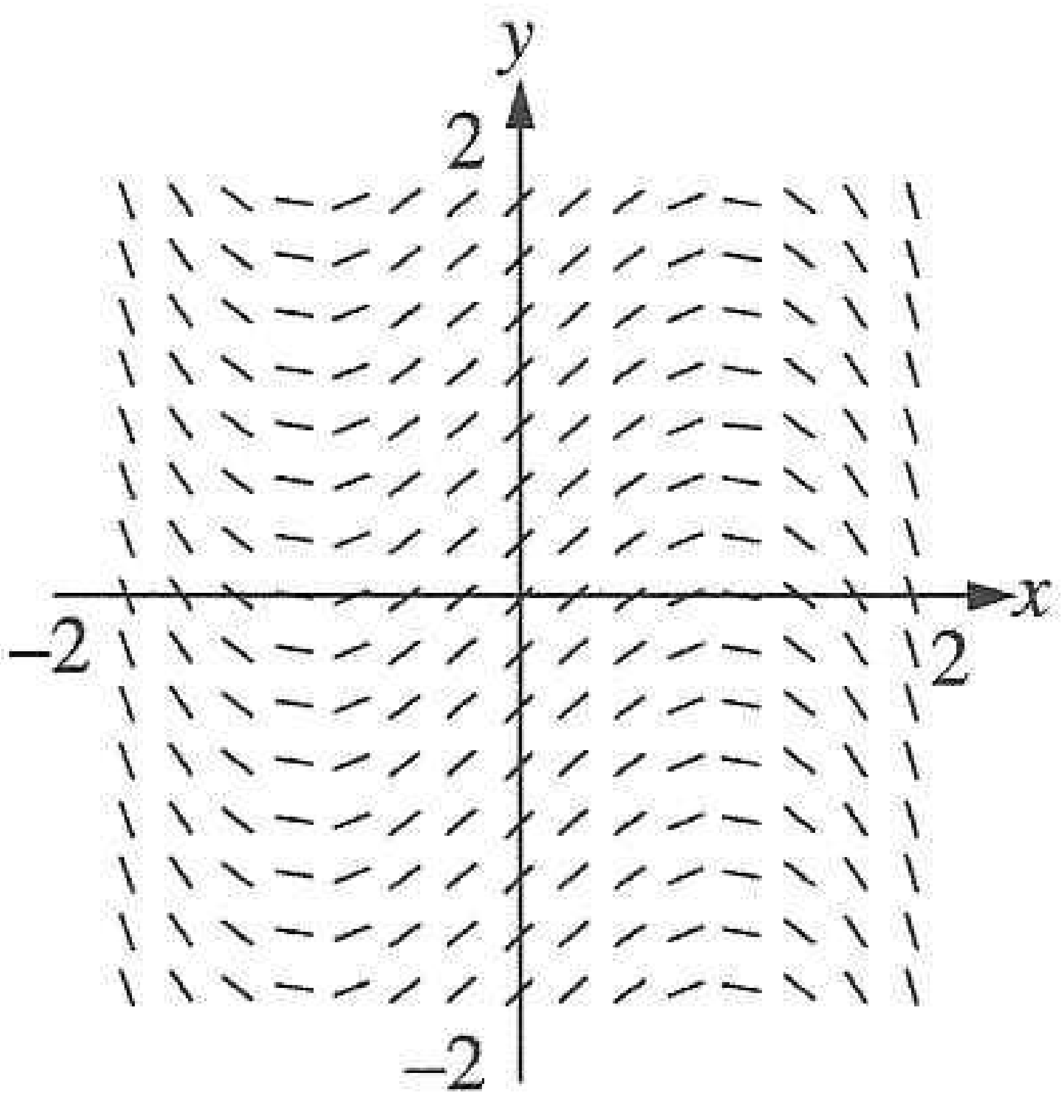
(A)



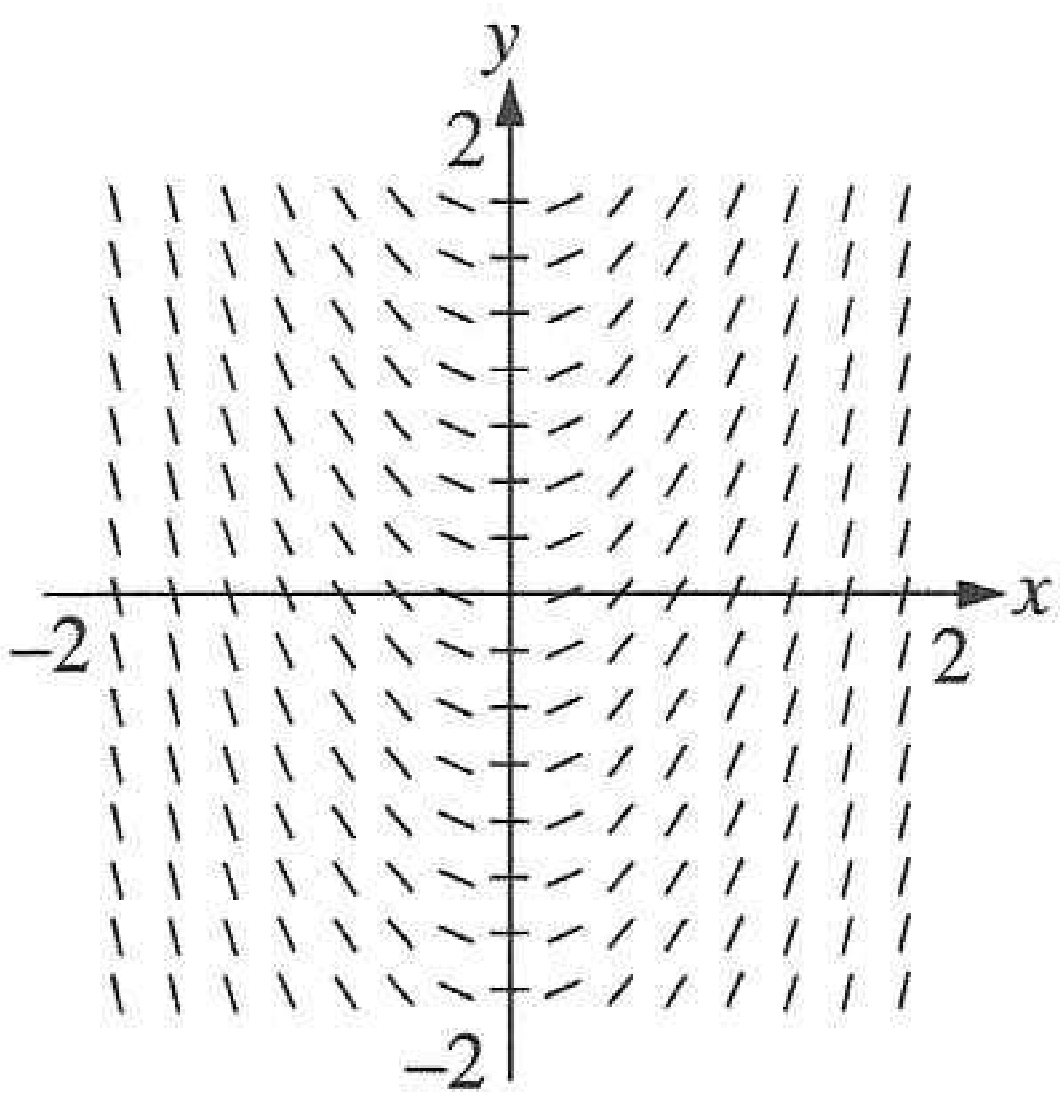
(B)



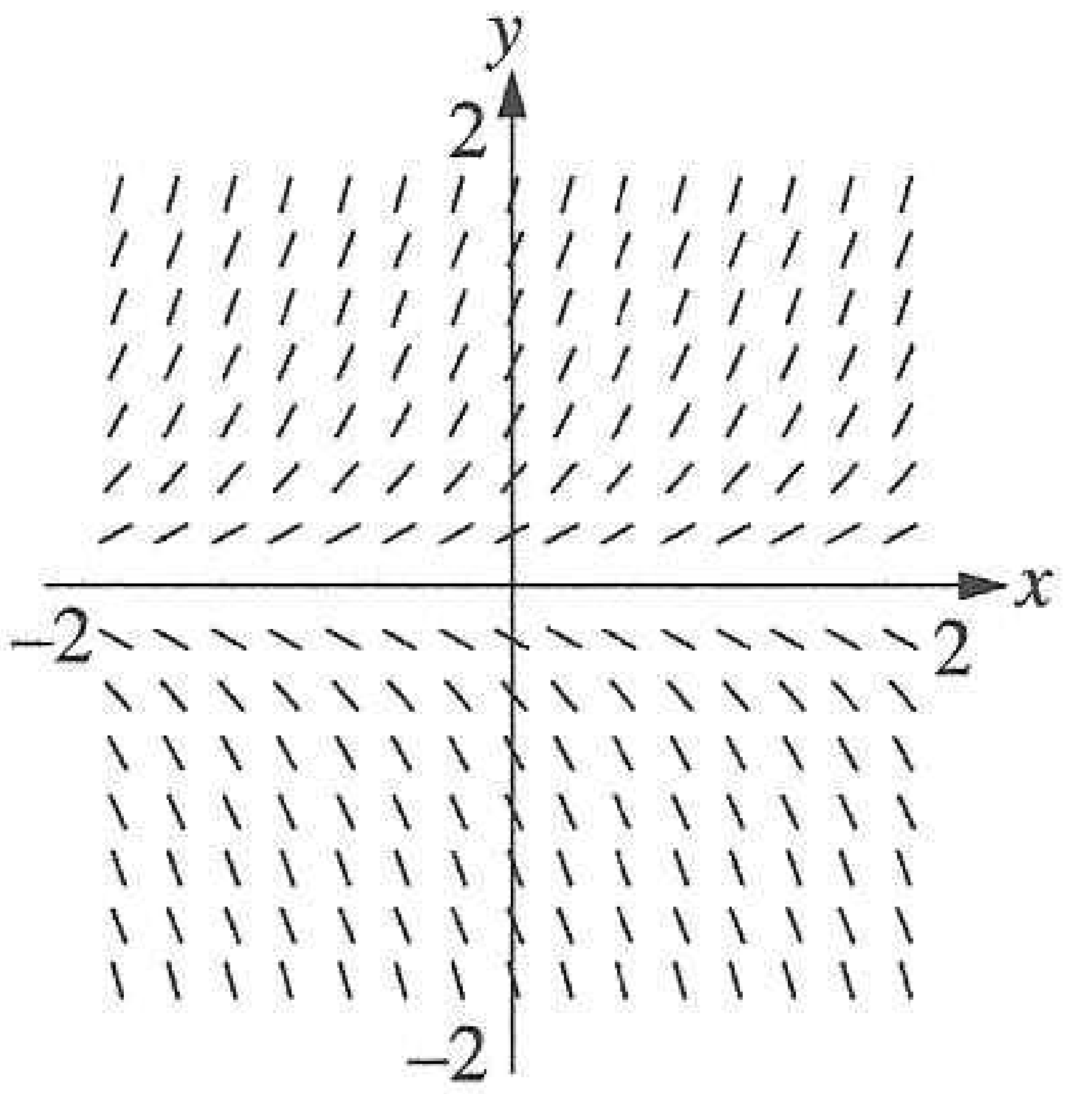
(C)



(D)



(E)



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28. In the *xy*-plane, a particle moves along the parabola $y = x^2 - x$ with a constant speed of $2\sqrt{10}$ units per second. If $\frac{dx}{dt} > 0$, what is the value of $\frac{dy}{dt}$ when the particle is at the point $(2, 2)$?

(A) $\frac{2}{3}$ (B) $\frac{2\sqrt{10}}{3}$ (C) 3 (D) 6 (E) $6\sqrt{10}$

END OF PART A OF SECTION I

CALCULUS BC
SECTION I, Part B
Time—50 minutes

Number of questions—17

A GRAPHING CALCULATOR IS REQUIRED FOR SOME QUESTIONS ON THIS PART OF THE EXAM.

Directions: Solve each of the following problems, using the available space for scratch work. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding oval on the answer sheet. No credit will be given for anything written in the exam book. Do not spend too much time on any one problem.

BE SURE YOU ARE USING PAGE 3 OF THE ANSWER SHEET TO RECORD YOUR ANSWERS TO QUESTIONS NUMBERED 76-92.

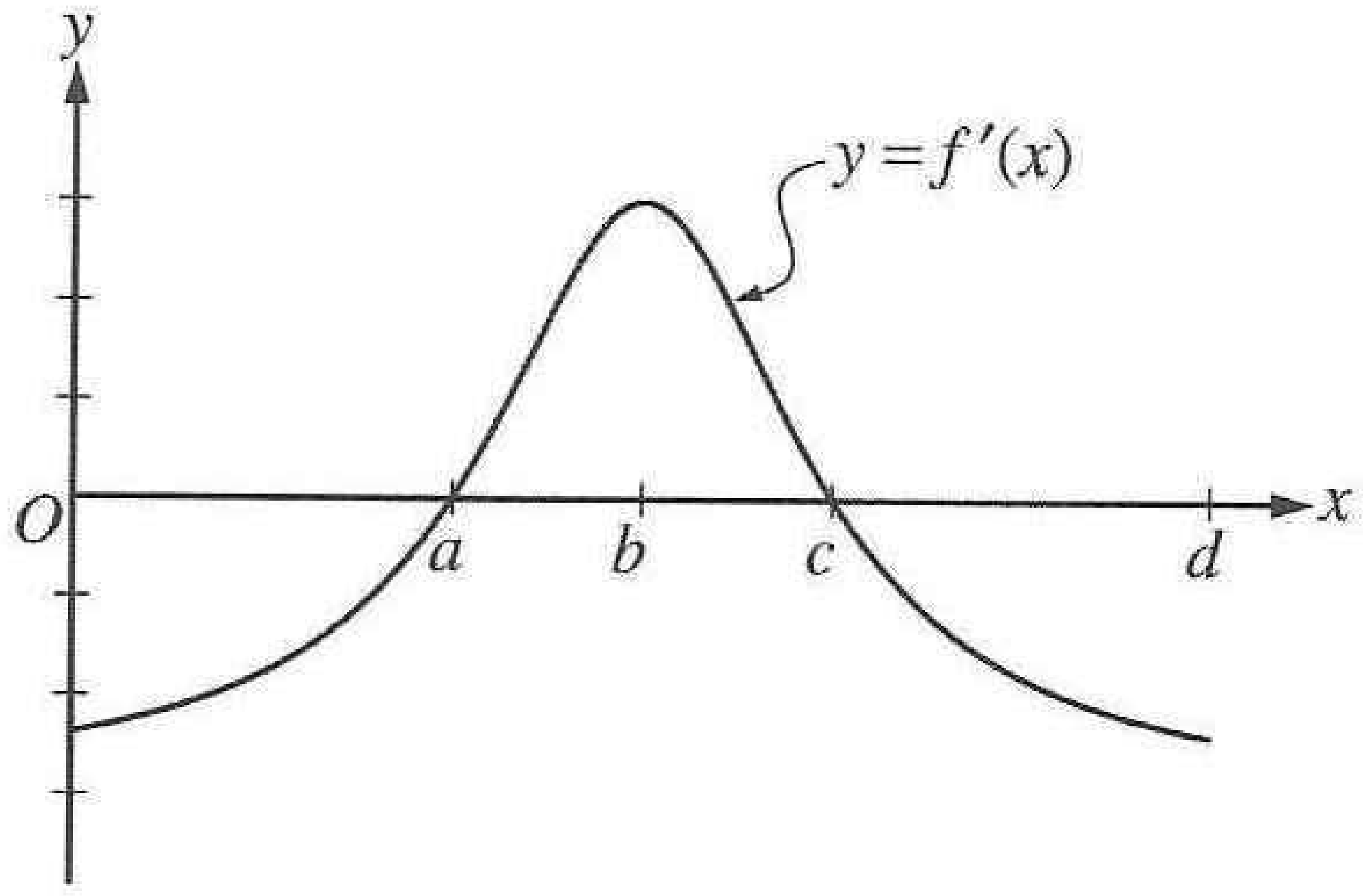
YOU MAY NOT RETURN TO PAGE 2 OF THE ANSWER SHEET.

In this exam:

(1) The exact numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.

(2) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

(3) The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1} x = \arcsin x$).



76. The graph of f', the derivative of a function f, is shown above. The domain of f is the open interval $0 < x < d$. Which of the following statements is true?

(A) f has a local minimum at $x = c$.

(B) f has a local maximum at $x = b$.

(C) The graph of f has a point of inflection at $(a, f(a))$.

(D) The graph of f has a point of inflection at $(b, f(b))$.

(E) The graph of f is concave up on the open interval (c, d).

---

77. Water is pumped out of a lake at the rate $R(t) = 12\sqrt{\frac{t}{t+1}}$ cubic meters per minute, where t is measured in minutes. How much water is pumped from time $t = 0$ to $t = 5$?

(A) 9.439 cubic meters

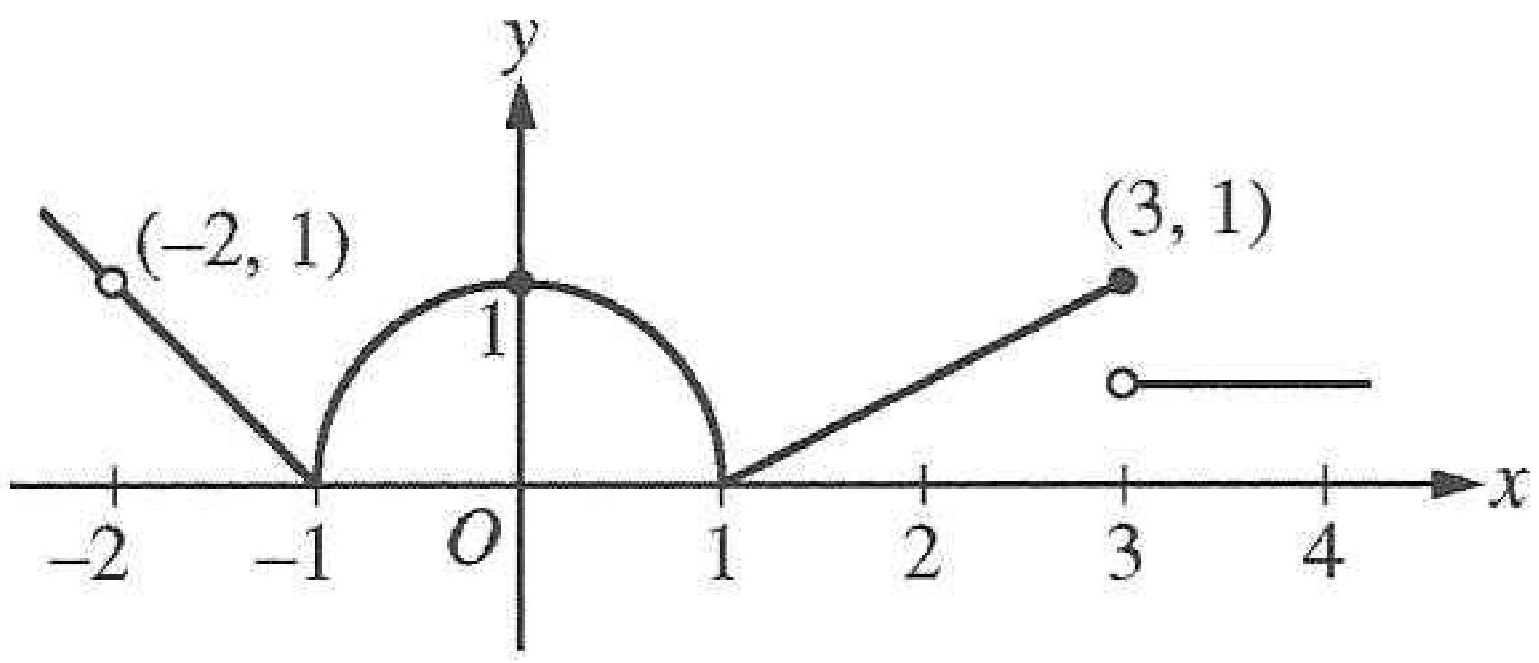
(B) 10.954 cubic meters

(C) 43.816 cubic meters

(D) 47.193 cubic meters

(E) 54.772 cubic meters

GO ON TO THE NEXT PAGE.



Graph of f

78. The graph of a function f is shown above. For which of the following values of c does $\lim\limits_{x \to c} f(x) = 1$?

(A) 0 only

(B) 0 and 3 only

(C) -2 and 0 only

(D) -2 and 3 only

(E) -2, 0, and 3

---

79. Let f be a positive, continuous, decreasing function such that $a_n = f(n)$. If $\sum\limits_{n=1}^{\infty} a_n$ converges to k, which of the following must be true?

(A) $\lim\limits_{n \to \infty} a_n = k$

(B) $\int_1^n f(x)\,dx = k$

(C) $\int_1^\infty f(x)\,dx$ diverges.

(D) $\int_1^\infty f(x)\,dx$ converges.

(E) $\int_1^\infty f(x)\,dx = k$

GO ON TO THE NEXT PAGE.

80. The derivative of the function f is given by $f'(x) = x^2 \cos\left(x^2\right)$. How many points of inflection does the graph of f have on the open interval $(-2, 2)$?

(A) One (B) Two (C) Three (D) Four (E) Five

81. Let f and g be continuous functions for $a \le x \le b$. If $a < c < b$, $\int_a^b f(x)\,dx = P$, $\int_c^b f(x)\,dx = Q$, $\int_a^b g(x)\,dx = R$, and $\int_c^b g(x)\,dx = S$, then $\int_a^c (f(x) - g(x))\,dx =$

(A) $P - Q + R - S$

(B) $P - Q - R + S$

(C) $P - Q - R - S$

(D) $P + Q - R - S$

(E) $P + Q - R + S$

GO ON TO THE NEXT PAGE.

82. If $\sum_{n=1}^{\infty} a_n$ diverges and $0 \le a_n \le b_n$ for all n, which of the following statements must be true?

(A) $\sum_{n=1}^{\infty} (-1)^n a_n$ converges.

(B) $\sum_{n=1}^{\infty} (-1)^n b_n$ converges.

(C) $\sum_{n=1}^{\infty} (-1)^n b_n$ diverges.

(D) $\sum_{n=1}^{\infty} b_n$ converges.

(E) $\sum_{n=1}^{\infty} b_n$ diverges.

---

83. What is the area enclosed by the curves $y = x^3 - 8x^2 + 18x - 5$ and $y = x + 5$?

(A) 10.667 (B) 11.833 (C) 14.583 (D) 21.333 (E) 32

84. Let f be a function with $f(3) = 2$, $f'(3) = -1$, $f''(3) = 6$, and $f'''(3) = 12$. Which of the following is the third-degree Taylor polynomial for f about $x = 3$?

(A) $2 - (x-3) + 3(x-3)^2 + 2(x-3)^3$

(B) $2 - (x-3) + 3(x-3)^2 + 4(x-3)^3$

(C) $2 - (x-3) + 6(x-3)^2 + 12(x-3)^3$

(D) $2 - x + 3x^2 + 2x^3$

(E) $2 - x + 6x^2 + 12x^3$

---

85. A particle moves on the x-axis with velocity given by $v(t) = 3t^4 - 11t^2 + 9t - 2$ for $-3 \le t \le 3$. How many times does the particle change direction as t increases from -3 to 3 ?

(A) Zero (B) One (C) Two (D) Three (E) Four

GO ON TO THE NEXT PAGE.

86. On the graph of $y = f(x)$, the slope at any point (x, y) is twice the value of x. If $f(2) = 3$, what is the value of $f(3)$?

(A) 6 (B) 7 (C) 8 (D) 9 (E) 10

---

87. An object traveling in a straight line has position $x(t)$ at time t. If the initial position is $x(0) = 2$ and the velocity of the object is $v(t) = \sqrt[3]{1 + t^2}$, what is the position of the object at time $t = 3$?

(A) 0.431 (B) 2.154 (C) 4.512 (D) 6.512 (E) 17.408

88. For all values of x, the continuous function f is positive and decreasing. Let g be the function given by $g(x) = \int_2^x f(t)\,dt$. Which of the following could be a table of values for g ?

(A)

x	$g(x)$
1	–2
2	0
3	1

(B)

x	$g(x)$
1	–2
2	0
3	3

(C)

x	$g(x)$
1	1
2	0
3	–2

(D)

x	$g(x)$
1	2
2	0
3	–1

(E)

x	$g(x)$
1	3
2	0
3	2

---

89. The function f is continuous for $-2 \le x \le 2$ and $f(-2) = f(2) = 0$. If there is no c, where $-2 < c < 2$, for which $f'(c) = 0$, which of the following statements must be true?

(A) For $-2 < k < 2$, $f'(k) > 0$.

(B) For $-2 < k < 2$, $f'(k) < 0$.

(C) For $-2 < k < 2$, $f'(k)$ exists.

(D) For $-2 < k < 2$, $f'(k)$ exists, but f' is not continuous.

(E) For some k, where $-2 < k < 2$, $f'(k)$ does not exist.

GO ON TO THE NEXT PAGE.

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
-1	-5	1	3	0
0	-2	0	1	1
1	0	-3	0	0.5
2	5	-1	5	2

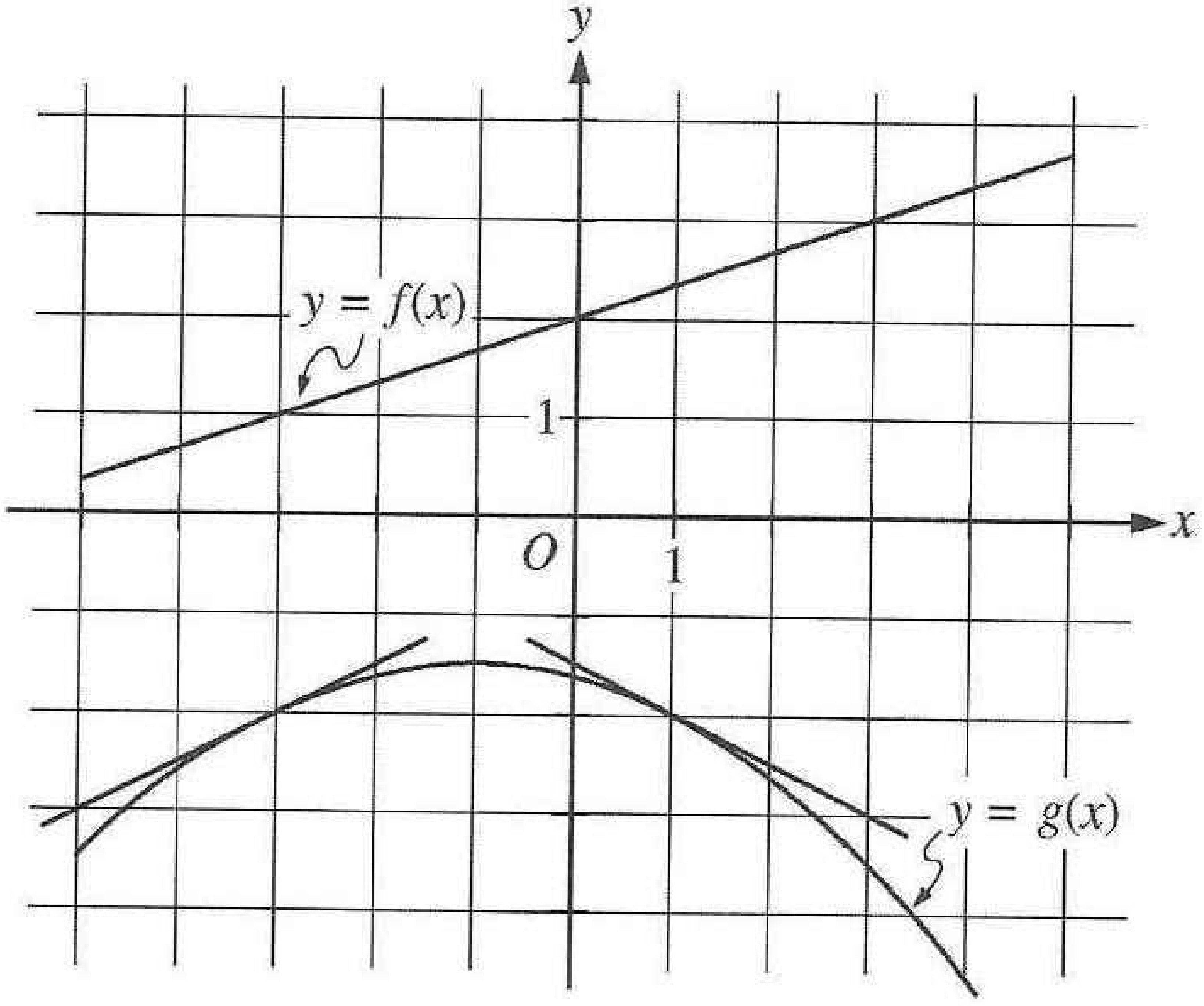
90. The table above gives values of the differentiable functions f and g and of their derivatives f' and g', at selected values of x. If $h(x) = f(g(x))$, what is the slope of the graph of h at $x = 2$?

(A) -10 (B) -6 (C) 5 (D) 6 (E) 10

---

91. Let f be the function given by $f(x) = \int_{1/3}^{x} \cos\left(\frac{1}{t^2}\right) dt$ for $\frac{1}{3} \le x \le 1$. At which of the following values of x does f attain a relative maximum?

(A) 0.357 and 0.798 (B) 0.4 and 0.564 (C) 0.4 only (D) 0.461 (E) 0.999



92. The figure above shows the graphs of the functions f and g. The graphs of the lines tangent to the graph of g at $x = -3$ and $x = 1$ are also shown. If $B(x) = g(f(x))$, what is $B'(-3)$?

(A) $-\frac{1}{2}$ (B) $-\frac{1}{6}$ (C) $\frac{1}{6}$ (D) $\frac{1}{3}$ (E) $\frac{1}{2}$

END OF SECTION I

AFTER TIME HAS BEEN CALLED, TURN TO THE NEXT PAGE AND ANSWER QUESTIONS 93-96.

93. Which graphing calculator did you use during the exam?

(A) Casio 6300, Casio 7300, Casio 7400, Casio 7700, TI-73, TI-80, or TI-81
(B) Casio 9700, Casio 9800, Sharp 9200, Sharp 9300, TI-82, or TI-85
(C) Casio 9750, Casio 9850, Casio 9860, Casio FX 1.0, Sharp 9600, Sharp 9900, TI-83, TI-83 Plus, TI-83 Plus Silver, TI-84 Plus, TI-84 Plus Silver, TI-86, or TI-Nspire
(D) Casio 9970, Casio Algebra FX 2.0, HP 38G, HP 39 series, HP 40G, HP 48 series, HP 49 series, HP 50 series, TI-89, TI-89 Titanium, or TI-Nspire CAS
(E) Some other graphing calculator

94. During your Calculus BC course, which of the following best describes your calculator use?

(A) I used my own graphing calculator.
(B) I used a graphing calculator furnished by my school, both in class and at home.
(C) I used a graphing calculator furnished by my school only in class.
(D) I used a graphing calculator furnished by my school mostly in class, but occasionally at home.
(E) I did not use a graphing calculator.

95. During your Calculus BC course, which of the following describes approximately how often a graphing calculator was used by you or your teacher in classroom learning activities?

(A) Almost every class
(B) About three-quarters of the classes
(C) About one-half of the classes
(D) About one-quarter of the classes
(E) Seldom or never

96. During your Calculus BC course, which of the following describes the portion of testing time you were allowed to use a graphing calculator?

(A) All or almost all of the time
(B) About three-quarters of the time
(C) About one-half of the time
(D) About one-quarter of the time
(E) Seldom or never

AP® Calculus BC Exam

SECTION II: Free-Response Questions

At a Glance

Total Time
1 hour, 30 minutes
Number of Questions
6
Percent of Total Grade
50%
Writing Instrument
Either pencil or pen with black or dark blue ink
Weight
The questions are weighted equally, but the parts of a question are not necessarily given equal weight.

Part A

Number of Questions
3
Time
45 minutes
Electronic Device
Graphing calculator required
Percent of Section II Score
50%

Part B

Number of Questions
3
Time
45 minutes
Electronic Device
None allowed
Percent of Section II Score
50%

Instructions

The questions for Part A are printed in the green insert and the questions for Part B are printed in the blue insert. You may use the inserts to organize your answers and for scratch work, but you must write your answers in the pink Section II booklet. No credit will be given for work written in the inserts. Write your solution to each part of each question in the space provided for that part in the Section II booklet. Write clearly and legibly. Cross out any errors you make; erased or crossed-out work will not be graded.

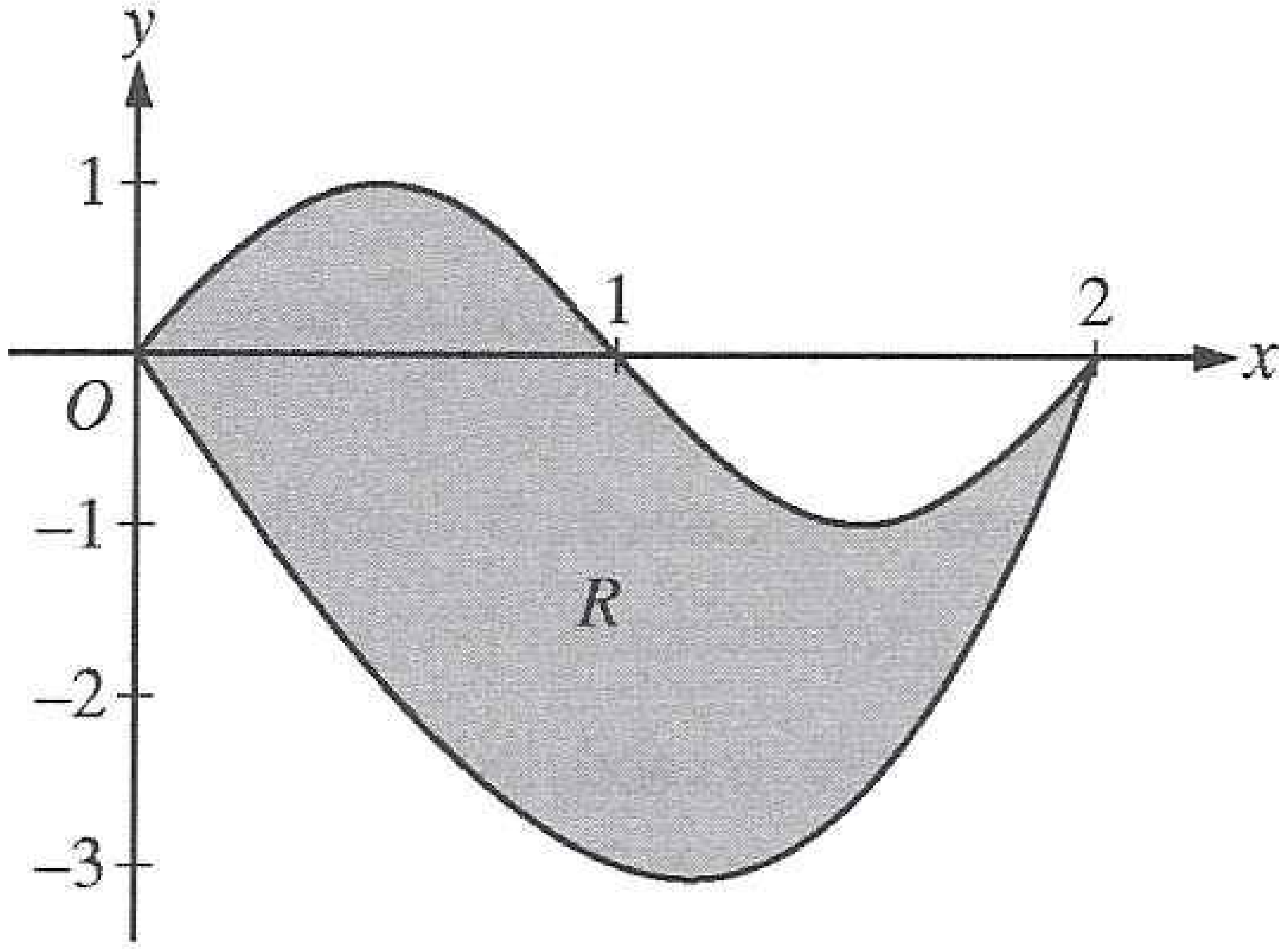
Manage your time carefully. During the timed portion for Part A, work only on the questions in Part A. You are permitted to use your calculator to solve an equation, find the derivative of a function at a point, or calculate the value of a definite integral. However, you must clearly indicate the setup of your question, namely the equation, function, or integral you are using. If you use other built-in features or programs, you must show the mathematical steps necessary to produce your results. During the timed portion for Part B, you may keep the green insert and continue to work on the questions in Part A without the use of a calculator.

For each part of Section II, you may wish to look over the questions before starting to work on them. It is not expected that everyone will be able to complete all parts of all questions.

- Show all of your work. Clearly label any functions, graphs, tables, or other objects that you use. Your work will be graded on the correctness and completeness of your methods as well as your answers. Answers without supporting work may not receive credit. Justifications require that you give mathematical (noncalculator) reasons.
- Your work must be expressed in standard mathematical notation rather than calculator syntax. For example, $\int_1^5 x^2\,dx$ may not be written as fnInt(X², X, 1, 5).
- Unless otherwise specified, answers (numeric or algebraic) need not be simplified. If you use decimal approximations in calculations, your work will be graded on accuracy. Unless otherwise specified, your final answers should be accurate to three places after the decimal point.
- Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

CALCULUS BC
SECTION II, Part A
Time—45 minutes
Number of problems—3

A graphing calculator is required for some problems or parts of problems.



1. Let R be the region bounded by the graphs of $y = \sin(\pi x)$ and $y = x^3 - 4x$, as shown in the figure above.

   (a) Find the area of R.

   (b) The horizontal line $y = -2$ splits the region R into two parts. Write, but do not evaluate, an integral expression for the area of the part of R that is below this horizontal line.

   (c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a square. Find the volume of this solid.

   (d) The region R models the surface of a small pond. At all points in R at a distance x from the y-axis, the depth of the water is given by $h(x) = 3 - x$. Find the volume of water in the pond.

t (hours)	0	1	3	4	7	8	9
$L(t)$ (people)	120	156	176	126	150	80	0

2. Concert tickets went on sale at noon $(t = 0)$ and were sold out within 9 hours. The number of people waiting in line to purchase tickets at time t is modeled by a twice-differentiable function L for $0 \le t \le 9$. Values of $L(t)$ at various times t are shown in the table above.

   (a) Use the data in the table to estimate the rate at which the number of people waiting in line was changing at 5:30 P.M. $(t = 5.5)$. Show the computations that lead to your answer. Indicate units of measure.

   (b) Use a trapezoidal sum with three subintervals to estimate the average number of people waiting in line during the first 4 hours that tickets were on sale.

   (c) For $0 \le t \le 9$, what is the fewest number of times at which $L'(t)$ must equal 0 ? Give a reason for your answer.

   (d) The rate at which tickets were sold for $0 \le t \le 9$ is modeled by $r(t) = 550te^{-t/2}$ tickets per hour. Based on the model, how many tickets were sold by 3 P.M. $(t = 3)$, to the nearest whole number?

x	$h(x)$	$h'(x)$	$h''(x)$	$h'''(x)$	$h^{(4)}(x)$
1	11	30	42	99	18
2	80	128	$\frac{488}{3}$	$\frac{448}{3}$	$\frac{584}{9}$
3	317	$\frac{753}{2}$	$\frac{1383}{4}$	$\frac{3483}{16}$	$\frac{1125}{16}$

3. Let h be a function having derivatives of all orders for $x > 0$. Selected values of h and its first four derivatives are indicated in the table above. The function h and these four derivatives are increasing on the interval $1 \leq x \leq 3$.

(a) Write the first-degree Taylor polynomial for h about $x = 2$ and use it to approximate $h(1.9)$. Is this approximation greater than or less than $h(1.9)$? Explain your reasoning.

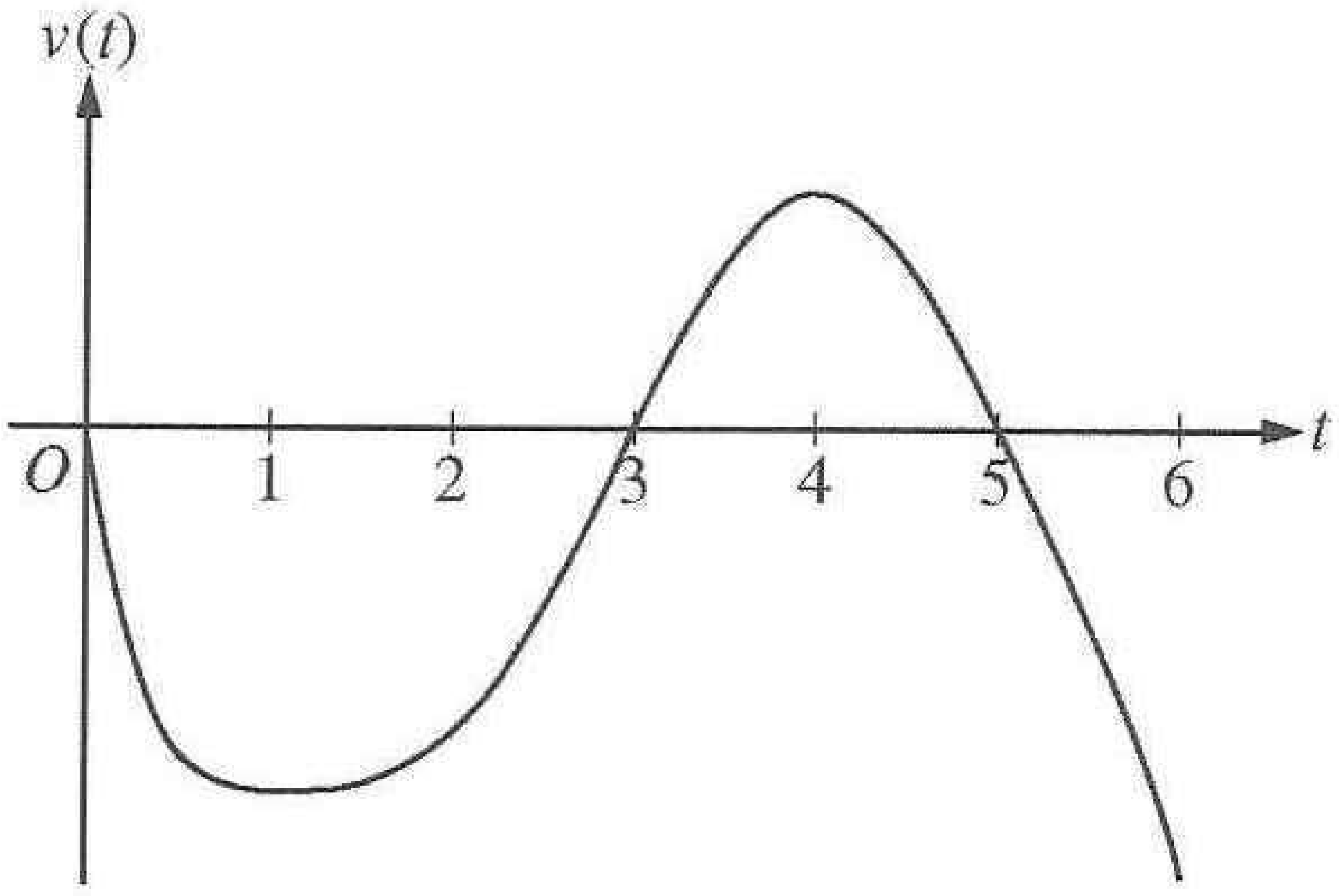
(b) Write the third-degree Taylor polynomial for h about $x = 2$ and use it to approximate $h(1.9)$.

(c) Use the Lagrange error bound to show that the third-degree Taylor polynomial for h about $x = 2$ approximates $h(1.9)$ with error less than 3×10^{-4}.

END OF PART A OF SECTION II

CALCULUS BC
SECTION II, Part B
Time—45 minutes
Number of problems—3

No calculator is allowed for these problems.



Graph of v

4. A particle moves along the x-axis so that its velocity at time t, for $0 \le t \le 6$, is given by a differentiable function v whose graph is shown above. The velocity is 0 at $t = 0$, $t = 3$, and $t = 5$, and the graph has horizontal tangents at $t = 1$ and $t = 4$. The areas of the regions bounded by the t-axis and the graph of v on the intervals $[0, 3]$, $[3, 5]$, and $[5, 6]$ are 8, 3, and 2, respectively. At time $t = 0$, the particle is at $x = -2$.

   (a) For $0 \le t \le 6$, find both the time and the position of the particle when the particle is farthest to the left. Justify your answer.

   (b) For how many values of t, where $0 \le t \le 6$, is the particle at $x = -8$? Explain your reasoning.

   (c) On the interval $2 < t < 3$, is the speed of the particle increasing or decreasing? Give a reason for your answer.

   (d) During what time intervals, if any, is the acceleration of the particle negative? Justify your answer.

---

5. The derivative of a function f is given by $f'(x) = (x - 3)e^x$ for $x > 0$, and $f(1) = 7$.

   (a) The function f has a critical point at $x = 3$. At this point, does f have a relative minimum, a relative maximum, or neither? Justify your answer.

   (b) On what intervals, if any, is the graph of f both decreasing and concave up? Explain your reasoning.

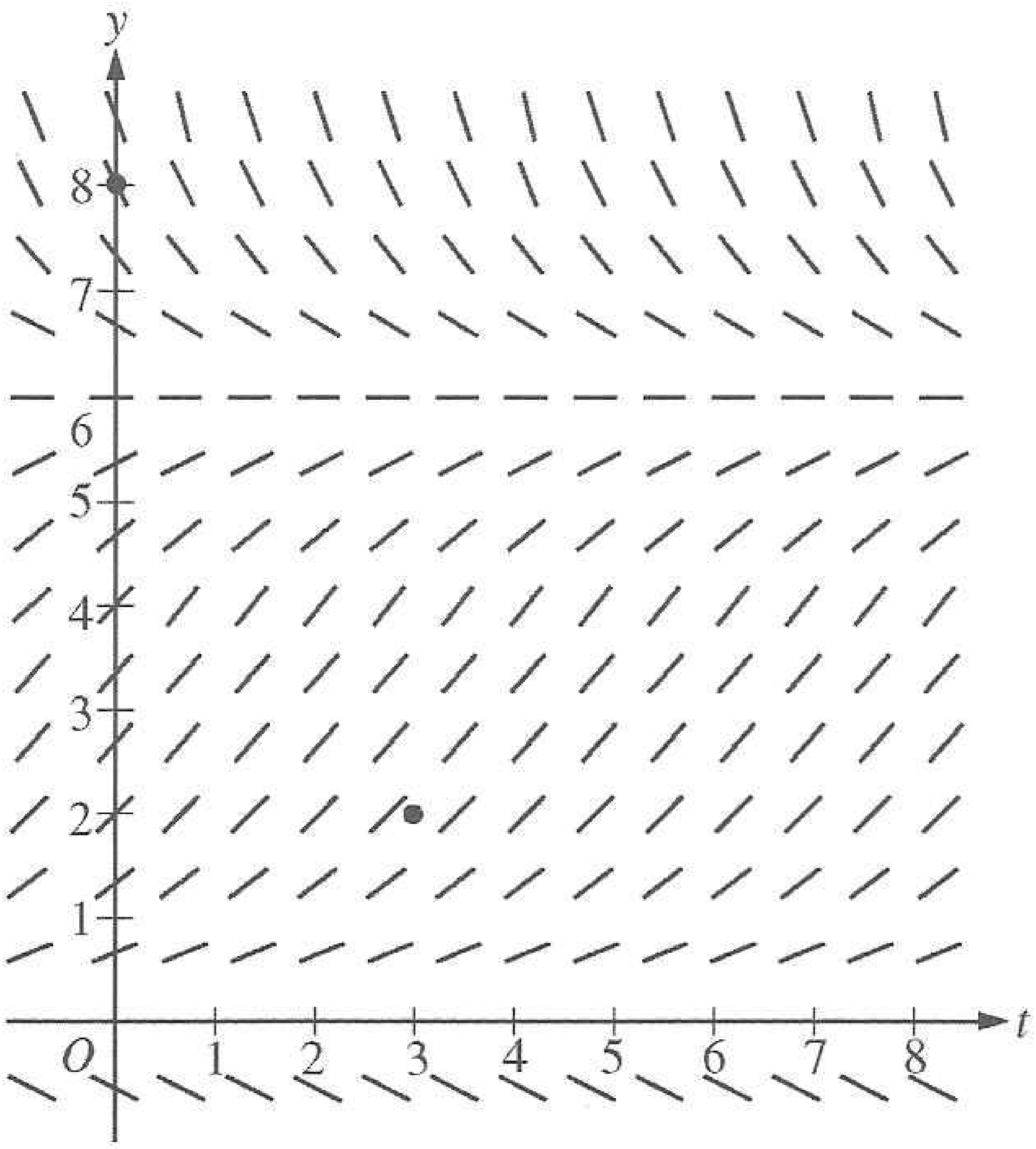
   (c) Find the value of $f(3)$.

---

6. Consider the logistic differential equation $\frac{dy}{dt} = \frac{y}{8}(6 - y)$. Let $y = f(t)$ be the particular solution to the differential equation with $f(0) = 8$.

(a) A slope field for this differential equation is given below. Sketch possible solution curves through the points $(3, 2)$ and $(0, 8)$.

(Note: Use the axes provided in the exam booklet.)



(b) Use Euler's method, starting at $t = 0$ with two steps of equal size, to approximate $f(1)$.

(c) Write the second-degree Taylor polynomial for f about $t = 0$, and use it to approximate $f(1)$.

(d) What is the range of f for $t \geq 0$?

END OF EXAM

Chapter III: Answers to the 2008 AP Calculus AB and Calculus BC Exams

- Section I: Multiple Choice
    - Section I Answer Key and Percent Answering Correctly
    - Analyzing Your Students' Performance on the Multiple-Choice Section
    - Diagnostic Guide for the 2008 AP Calculus AB and Calculus BC Exams
- Section II: Free Response
    - Comments from the Chief Reader
    - Commentary, Scoring Guidelines, and Sample Student Responses
        - AB/BC Question 1
        - AB/BC Question 2
        - AB Question 3
        - AB/BC Question 4
        - AB Question 5
        - AB Question 6
        - BC Question 3
        - BC Question 5
        - BC Question 6

Section I: Multiple Choice

On the following pages are the correct answers to the multiple-choice questions for Form Q, the percent of AP students who answered each question correctly by AP grade, and the total percent answering correctly.

Section I Answer Key and Percent Answering Correctly
Calculus AB

Item No.	Correct Answer	Percent Correct by Grade					Total Percent Correct
		5	4	3	2	1	
1	B	88	77	68	58	36	65
2	D	96	84	71	57	32	67
3	D	72	48	35	26	14	39
4	B	95	86	74	60	37	70
5	A	74	59	49	40	24	49
6	A	74	55	46	39	30	49
7	B	97	89	76	55	22	67
8	E	86	74	64	52	29	61
9	D	95	82	65	43	16	60
10	C	90	79	69	58	35	66
11	B	98	92	82	69	44	77
12	D	89	79	69	57	33	65
13	A	72	51	36	24	14	40
14	E	50	30	25	23	18	29
15	C	93	82	72	61	41	70
16	D	74	46	30	20	11	37
17	C	86	62	43	31	26	50
18	A	47	23	15	12	11	22
19*	–	–	–	–	–	–	–
20	D	73	50	35	25	19	41
21	A	48	26	17	11	7	22
22	B	80	64	58	55	50	62
23	E	50	26	17	12	7	23
24	B	80	54	33	18	9	40
25	B	55	26	16	14	13	26
26	A	53	29	17	12	9	25
27	C	70	52	41	31	20	43
28	A	38	13	6	3	3	14
76	B	98	93	83	65	30	73
77	C	89	73	56	41	24	57
78	B	97	91	80	61	28	71
79	B	81	66	54	44	28	55
80	E	70	52	38	26	14	40
81	E	90	71	55	42	21	56
82	B	96	88	77	61	27	69
83	B	90	76	63	48	27	61
84	C	91	80	64	41	14	58
85	B	76	53	38	28	18	43
86	C	86	71	58	43	18	55
87	D	84	55	35	21	11	42
88	C	96	84	65	45	21	62
89	E	67	40	23	16	10	32
90	A	67	37	20	11	10	30
91	C	92	81	68	52	27	64
92	B	58	45	43	41	36	45

*Although 45 multiple-choice items were administered in Section I, item # 19 was not used in scoring for statistical reasons. Many students who scored well on the exam chose option A or option C, instead of the correct answer E. It is important for teachers to emphasize that when determining the horizontal asymptotes for the graph of $y = f(x)$, students must examine the limit of $f(x)$ as x approaches infinity and the limit of $f(x)$ as x approaches negative infinity.

Section I Answer Key and Percent Answering Correctly
Calculus BC

Item No.	Correct Answer	Percent Correct by Grade					Total Percent Correct
		5	4	3	2	1	
1	B	97	95	93	89	81	93
2	A	96	92	87	80	67	89
3	C	88	77	67	55	36	73
4	D	95	86	74	61	40	80
5	C	94	85	76	66	40	80
6	A	81	61	52	43	35	63
7	D	90	73	58	43	23	69
8	B	89	71	56	39	26	68
9	D	96	85	74	58	33	79
10	B	90	80	72	59	39	76
11	A	75	48	34	27	21	52
12	D	64	35	25	21	16	42
13	B	75	55	40	29	23	55
14	E	63	39	32	27	22	45
15	A	85	63	45	31	17	61
16	D	62	38	27	21	16	42
17	B	79	60	45	37	28	59
18	A	59	33	23	16	13	38
19	A	61	41	30	23	18	43
20	C	49	23	16	12	9	30
21	A	58	35	23	16	10	38
22	E	31	11	7	6	5	18
23	E	80	57	42	28	15	56
24	A	63	37	25	19	13	41
25	B	57	27	19	13	12	35
26	D	56	35	25	19	13	38
27	A	83	66	52	41	28	64
28	D	26	5	3	2	2	13
76	D	93	84	72	56	36	77
77	D	99	98	94	87	69	93
78	C	94	81	66	49	26	74
79	D	70	45	36	27	21	49
80	E	70	56	44	33	21	54
81	B	84	68	56	41	24	65
82	E	93	78	65	51	37	75
83	B	93	82	71	59	39	77
84	A	95	86	73	54	28	78
85	C	89	74	62	49	35	71
86	C	73	42	28	19	13	47
87	D	91	68	48	31	18	65
88	A	85	60	43	29	16	59
89	E	79	55	38	24	14	54
90	D	91	79	62	44	22	71
91	D	85	66	47	29	15	61
92	B	90	73	54	33	18	67

Analyzing Your Students' Performance on the Multiple-Choice Section

If you give your students the 2008 Calculus AB or Calculus BC Exam for practice, you may want to analyze the results to find overall strengths and weaknesses in their understanding of calculus. The following diagnostic worksheets will help you do this. You are permitted to photocopy and distribute them to your students for completion.

1. In each category, students should insert a check mark for each correct answer.
2. Add together the total number of correct answers for each category.
3. To compare the student's number of correct answers for a given category with the average number correct for that category, copy the number of correct answers to the "Number Correct" table at the end of the Diagnostic Guide.

In addition, under each item, the percent of AP students who answered correctly is shown, so students can analyze their performance on individual items. This information will be helpful in deciding how students should plan their study time. Please note that one item may appear in several different categories, as questions can cover different topics.

Diagnostic Guide for the 2008 AP Calculus AB Exam

Differential Calculus (Average number correct = 13.0)

Question #	1	3	5	6	8	11	12	13	14	16	18	20	21	22	24	25	26	27	28	76	77	78	80	82	84	88	89	90
Correct/Incorrect																												
Percent of Students Answering Correctly	*65*	*39*	*49*	*49*	*61*	*77*	*65*	*40*	*29*	*37*	*22*	*41*	*22*	*62*	*40*	*26*	*25*	*43*	*14*	*73*	*57*	*71*	*40*	*69*	*58*	*62*	*32*	*30*

Integral Calculus (Average number correct = 8.9)

Question #	2	4	7	9	10	15	17	23	79	81	83	85	86	87	91	92
Correct/Incorrect																
Percent of Students Answering Correctly	*67*	*70*	*67*	*60*	*66*	*70*	*50*	*23*	*55*	*56*	*61*	*43*	*55*	*42*	*64*	*45*

Part A—No Calculator (Average number correct = 12.8)

Question #	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	20	21	22	23	24	25	26	27	28
Correct/Incorrect																											
Percent of Students Answering Correctly	*65*	*67*	*39*	*70*	*49*	*49*	*67*	*61*	*60*	*66*	*77*	*65*	*40*	*29*	*70*	*37*	*50*	*22*	*41*	*22*	*62*	*23*	*40*	*26*	*25*	*43*	*14*

Part B—With Graphing Calculator (Average number correct = 9.1)

Question #	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92
Correct/Incorrect																	
Percent of Students Answering Correctly	*73*	*57*	*71*	*55*	*40*	*56*	*69*	*61*	*58*	*43*	*55*	*42*	*62*	*32*	*30*	*64*	*45*

Calculator-Active Items (Average number correct = 3.5)

Question #	78	80	82	83	87	91
Correct/Incorrect						
Percent of Students Answering Correctly	*71*	*40*	*69*	*61*	*42*	*64*

Diagnostic Guide for the 2008 AP Calculus AB Exam *(continued)*

Number Correct

	Differential Calculus	Integral Calculus	Part A—No Calculator	Part B—with Graphing Calculator	Calculator-Active Items
Number of Questions	28	16	27	17	6
Average Number Correct	13.0 (46.4%)	8.9 (55.6%)	12.8 (47.4%)	9.1 (53.5%)	3.5 (58.3%)
My Number Correct					

Diagnostic Guide for the 2008 AP Calculus BC Exam

Calculus AB Topics (Average number correct = 17.0)

Question #	2	6	8	9	10	13	14	15	17	18	21	25	27	76
Correct/Incorrect														
Percent of Students Answering Correctly	*89*	*63*	*68*	*79*	*76*	*55*	*45*	*61*	*59*	*38*	*38*	*35*	*64*	*77*

Question #	77	78	80	81	83	85	86	87	88	89	90	91	92
Correct/Incorrect													
Percent of Students Answering Correctly	*93*	*74*	*54*	*65*	*77*	*71*	*47*	*65*	*59*	*54*	*71*	*61*	*67*

Calculus BC Only Topics (Average number correct = 9.7)

Question #	1	3	4	5	7	11	12	16	19	20	22	23	24	26	28	79	82	84
Correct/Incorrect																		
Percent of Students Answering Correctly	*93*	*73*	*80*	*80*	*69*	*52*	*42*	*42*	*43*	*30*	*18*	*56*	*41*	*38*	*13*	*49*	*75*	*78*

Differential Calculus (Average number correct = 12.6)

Question #	1	3	6	7	10	14	15	17	18	21	25	27	28	76	78	80	85	89	90	91	92
Correct/Incorrect																					
Percent of Students Answering Correctly	*93*	*73*	*63*	*69*	*76*	*45*	*61*	*59*	*38*	*38*	*35*	*64*	*13*	*77*	*74*	*54*	*71*	*54*	*71*	*61*	*67*

Diagnostic Guide for the 2008 AP Calculus BC Exam *(continued)*

Integral Calculus (Average number correct = 9.7)

Question #	2	5	8	9	11	13	19	22	24	26	77	81	83	86	87	88
Correct/Incorrect																
Percent of Students Answering Correctly	*89*	*80*	*68*	*79*	*52*	*55*	*43*	*18*	*41*	*38*	*93*	*65*	*77*	*47*	*65*	*59*

Part A—No Calculator (Average number correct = 15.4)

Question #	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Correct/Incorrect														
Percent of Students Answering Correctly	*93*	*89*	*73*	*80*	*80*	*63*	*69*	*68*	*79*	*76*	*52*	*42*	*55*	*45*

Question #	15	16	17	18	19	20	21	22	23	24	25	26	27	28
Correct/Incorrect														
Percent of Students Answering Correctly	*61*	*42*	*59*	*38*	*43*	*30*	*38*	*18*	*56*	*41*	*35*	*38*	*64*	*13*

Part B—With Graphing Calculator (Average number correct = 11.4)

Question #	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92
Correct/Incorrect																	
Percent of Students Answering Correctly	*77*	*93*	*74*	*49*	*54*	*65*	*75*	*77*	*78*	*71*	*47*	*65*	*59*	*54*	*71*	*61*	*67*

Series Items (Average number correct = 4.5)

Question #	4	12	16	20	23	79	82	84
Correct/Incorrect								
Percent of Students Answering Correctly	*80*	*42*	*42*	*30*	*56*	*49*	*75*	*78*

Calculator-Active Items (Average number correct = 4.2)

Question #	77	80	83	85	87	91
Correct/Incorrect						
Percent of Students Answering Correctly	*93*	*54*	*77*	*71*	*65*	*61*

Diagnostic Guide for the 2008 AP Calculus BC Exam *(continued)*

Number Correct

	Calculus AB Topics	Calculus BC Only Topics	Differential Calculus	Integral Calculus	Part A—No Calculator	Part B—With Graphing Calculator	Series Items	Calculator-Active Items
Number of Questions	27	18	21	16	28	17	8	6
Average Number Correct	17.0 (63.0%)	9.7 (53.9%)	12.6 (60.0%)	9.7 (60.6%)	15.4 (55.0%)	11.4 (67.1%)	4.5 (56.3%)	4.2 (70.0%)
My Number Correct								

Section II: Free Response

Comments from the Chief Reader

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The scoring guidelines for the 2008 free-response questions were drafted by the Chief Reader and refined by Exam Leaders, Question Leaders, and their assistants. The scoring guidelines were given final approval by the Chief Reader only after test-scoring thousands of papers. As a result, the scoring guidelines were clear and easy to apply, and all questions read smoothly. Backreading (or rereading) by Table Leaders showed very consistent scoring by Readers. Statistical analysis of the multiple-choice questions showed both the 2008 Calculus AB and Calculus BC populations to be stronger in preparation than their counterparts in the previous year, but comparable in strength of preparation to their counterparts in years prior to 2007.

The result of the Calculus AB subscore grade for the Calculus BC population was consistent with the strong and reliable results from previous years. For the 2008 Calculus BC Exam, 98.5 percent of the AB subscore grades were at or above the level of the corresponding BC grade (66.4 percent of the AB subscore grades agreed with the BC grades; 32.1 percent of the AB subscore grades were higher than the BC grades). Only 1.5 percent of AB subscore grades were one grade lower than the BC grades. Almost all of the latter students, however, fall into the group that received a BC grade of 5 but an AB subscore of 4, which is still a very strong group. These results are close to what one would expect in consistency.

Comments on student performance on each free-response question are included on the following pages.

Commentary, Scoring Guidelines, and Sample Student Responses

The answers presented on the following pages are actual student responses to the free-response questions on the 2008 AP Calculus AB and Calculus BC Exams. The students gave permission to have their work reproduced at the time they took the exam. These responses were read and scored by the Table Leaders and Readers assigned to each particular question and were used as sample responses for the training of Readers during the AP Reading in June 2008. The actual scores that these students received, as well as a brief explanation of why, are indicated.

Calculus AB/BC Question 1

Overview

In this problem, students were given the graph of a region R bounded by two curves in the xy-plane. The points of intersection of the two curves were observable from the supplied graph. The formulas for the curves were given—a trigonometric function and a cubic polynomial—and students needed to match the appropriate functions to the upper and lower bounding curves. In each part, students had to set up and evaluate an appropriate integral. Part (a) asked for the area of R. Part (b) asked for the area of the portion of R below the line $y = -2$, so students needed to use a calculator to solve for the x-coordinates of the points of intersection of $y = -2$ and the lower curve to set up the appropriate integral. Part (c) asked for the volume of a solid with base R whose cross sections perpendicular to the x-axis are squares. In part (d) students were asked to find a volume in an applied setting. They had to determine that cross sections perpendicular to the x-axis are rectangles with one dimension in region R and the other dimension supplied by $h(x) = 3 - x$.

The mean score for this question was 4.89 for AB students and 6.38 for BC students. In part (a) students should realize that there is no need to partition the region R, even though the x-axis passes through the region. In parts (a), (b), and (c), students can and should use a graphing calculator to evaluate their integrals. Students who attempt antidifferentiation still need to arrive at a numerical answer to earn credit for their answer. Decimal answers without supporting work received no credit on this problem, even if the answer was correct. It is critical that students show their work on the free-response section of the exam. This includes showing the mathematical setup of a definite integral whose value is computed using the calculator.

Commentary on Student Responses

Student Response 1 (Score: 9)

The student earned all 9 points.

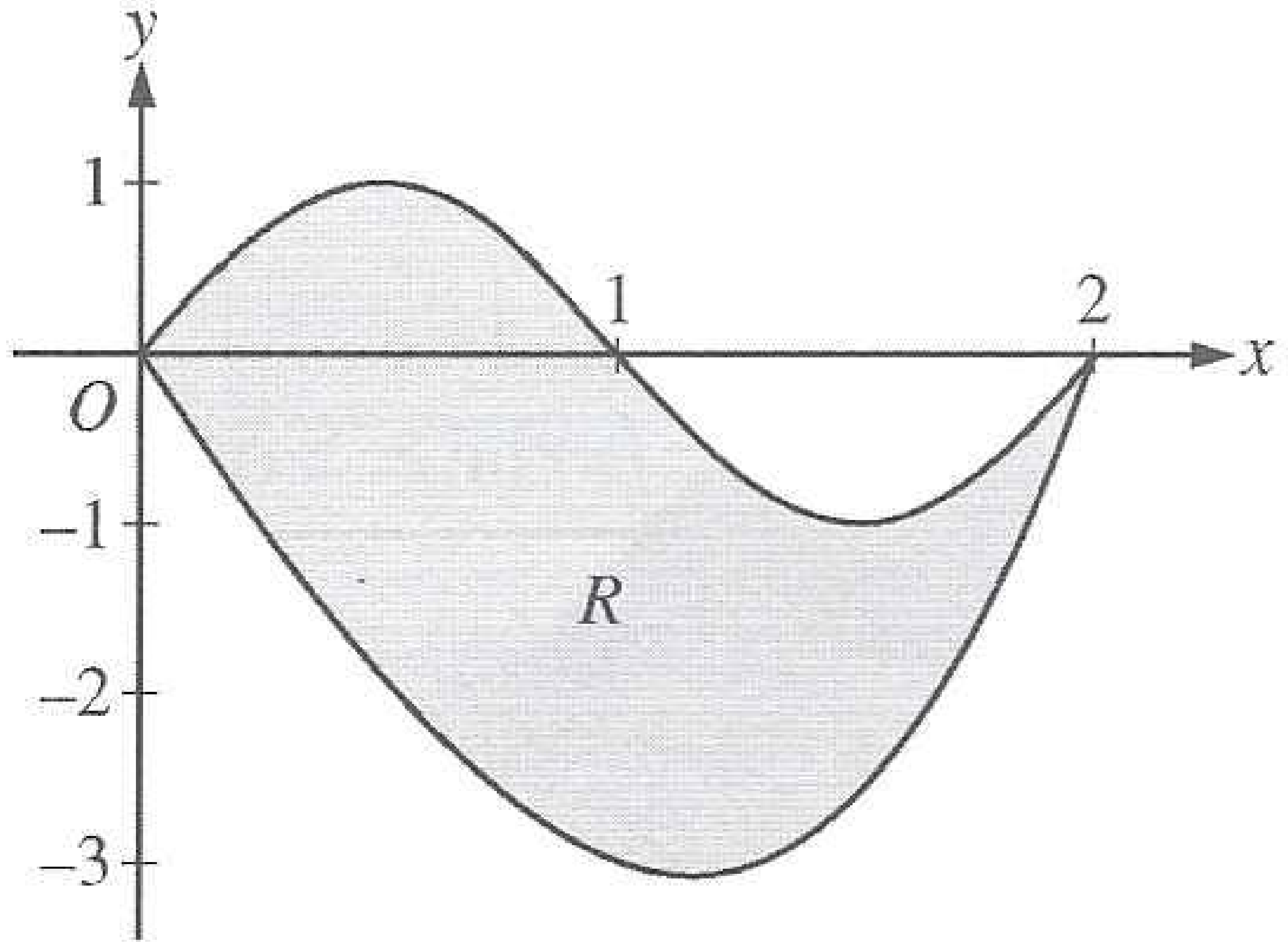
Student Response 2 (Score: 6)

The student earned 6 points: 3 points in part (a), 2 points in part (b), no points in part (c), and 1 point in part (d). In part (a) the student has the correct limits, integrand, and answer and earned all 3 points. In part (b) the limits are correct, as is the integrand, and the student earned both points. In part (c) the student does not present a correct integrand and did not earn the integrand point or the answer point. In part (d) the integrand is correct. The student multiplies the integral by 2π, so the answer is not correct.

Student Response 3 (Score: 4)

The student earned 4 points: 3 points in part (a), 1 point in part (b), no points in part (c), and no points in part (d). In part (a) the student does correct antidifferentiation without the use of the graphing calculator. In part (b) the student earned the integrand point. The lower limit is not correct, and so the limits point was not earned. In part (c) the integrand is not squared, and so the student did not earn the integrand point. The student is not eligible for the answer point. In part (d) the student does not present an integral and is not eligible for the answer point.

Scoring Guidelines for Calculus AB/BC Question 1



Let R be the region bounded by the graphs of $y = \sin(\pi x)$ and $y = x^3 - 4x$, as shown in the figure above.

(a) Find the area of R.

(b) The horizontal line $y = -2$ splits the region R into two parts. Write, but do not evaluate, an integral expression for the area of the part of R that is below this horizontal line.

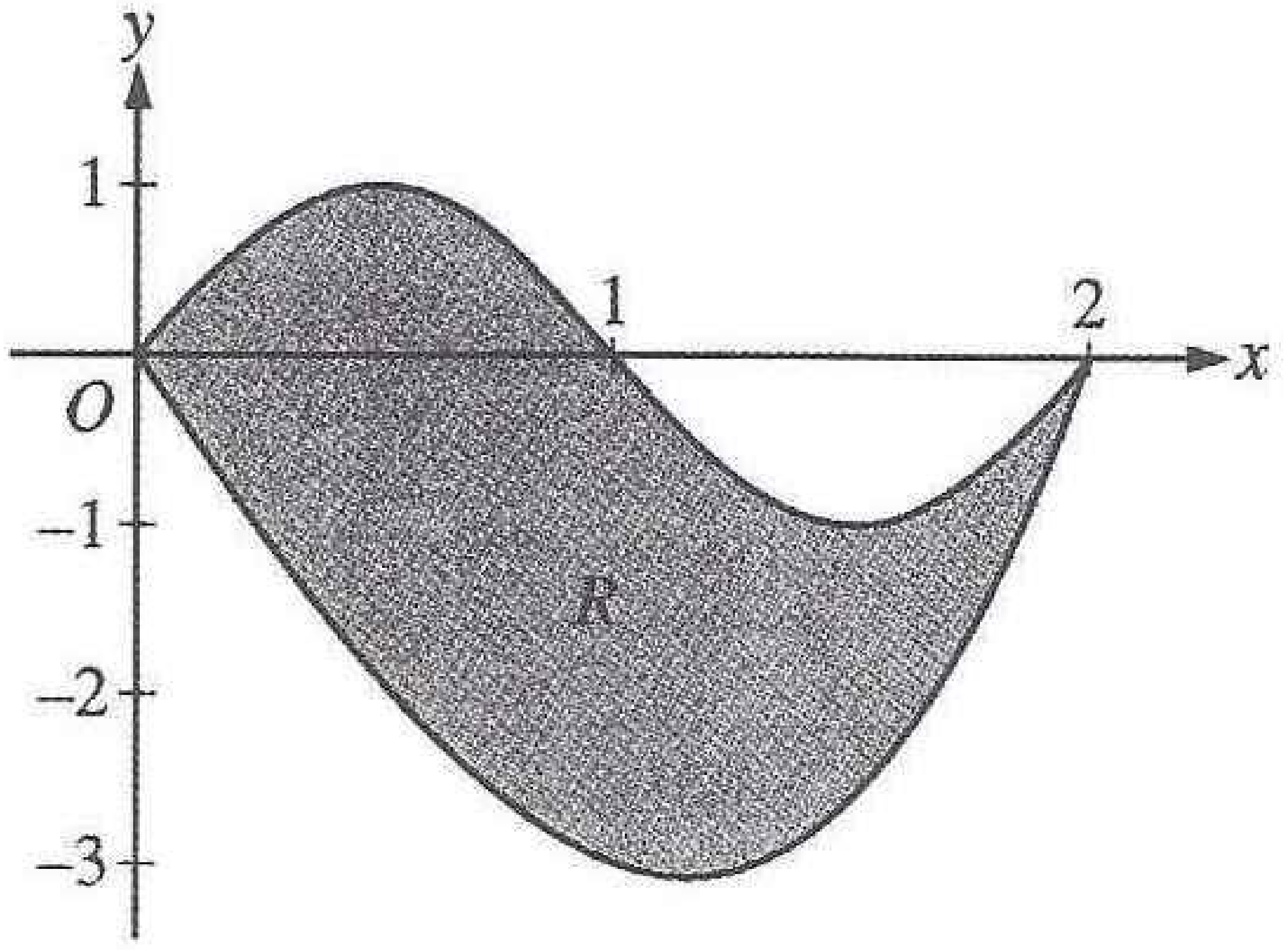
(c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a square. Find the volume of this solid.

(d) The region R models the surface of a small pond. At all points in R at a distance x from the y-axis, the depth of the water is given by $h(x) = 3 - x$. Find the volume of water in the pond.

(a) $\sin(\pi x) = x^3 - 4x$ at $x = 0$ and $x = 2$ Area $= \int_0^2 \left(\sin(\pi x) - \left(x^3 - 4x\right)\right) dx = 4$	$3 : \left\{ \begin{array}{l} 1 : \text{limits} \\ 1 : \text{integrand} \\ 1 : \text{answer} \end{array} \right.$
(b) $x^3 - 4x = -2$ at $r = 0.5391889$ and $s = 1.6751309$ The area of the stated region is $\int_r^s \left(-2 - \left(x^3 - 4x\right)\right) dx$	$2 : \left\{ \begin{array}{l} 1 : \text{limits} \\ 1 : \text{integrand} \end{array} \right.$
(c) Volume $= \int_0^2 \left(\sin(\pi x) - \left(x^3 - 4x\right)\right)^2 dx = 9.978$	$2 : \left\{ \begin{array}{l} 1 : \text{integrand} \\ 1 : \text{answer} \end{array} \right.$
(d) Volume $= \int_0^2 (3 - x)\left(\sin(\pi x) - \left(x^3 - 4x\right)\right) dx = 8.369$ or 8.370	$2 : \left\{ \begin{array}{l} 1 : \text{integrand} \\ 1 : \text{answer} \end{array} \right.$

Sample Student Responses for Calculus AB/BC Question 1

Student Response 1 (Score: 9)



Work for problem 1(a)

$R = \int \text{Top} - \text{Bottom}\, dx$

$R = \int_0^2 (\sin(\pi x)) - (x^3 - 4x)\, dx$

$\boxed{R = 4 \text{ units}^2}$

Work for problem 1(b)

$x^3 - 4x = 2$

$x = 0.539, 1.675$

$\boxed{R_B = \int_{0.539}^{1.675} (-2 - (x^3 - 4x))\, dx}$

Student Response 1 *(continued)*

Work for problem 1(c)

$A = s^2$

$s = T - B = \sin(\pi x) - (x^3 - 4x)$

$V_{cs} = \int_0^2 A\,dx = \int_0^2 (\sin(\pi x) - (x^3 - 4x))^2 dx$

$\boxed{= 9.978 \text{ units}^3}$

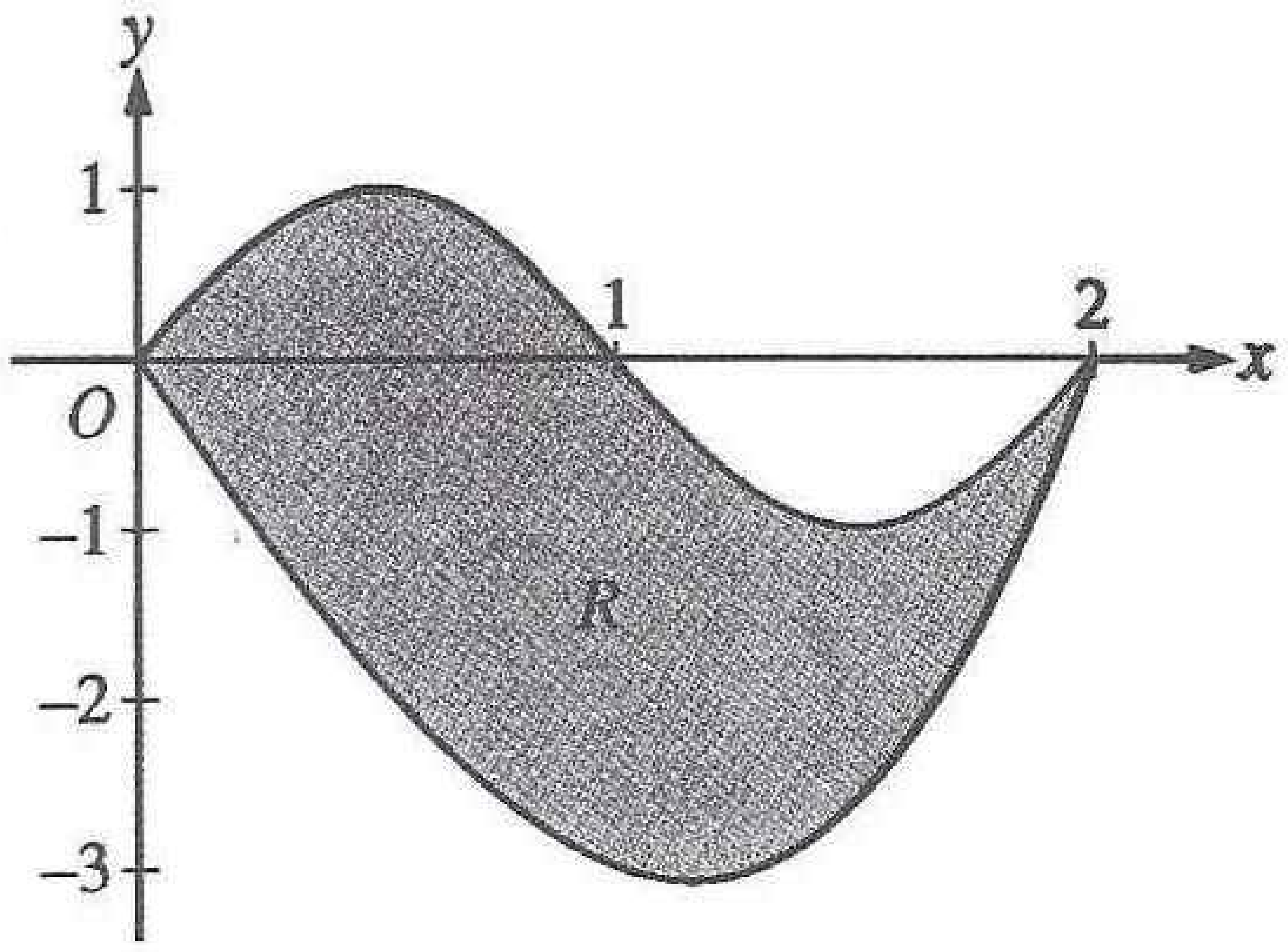
Work for problem 1(d)

$V = \int_0^2 A \cdot h$

$V = \int_0^2 (\sin(\pi x) - (x^3 - 4x)) \cdot (3 - x)\,dx$

$\boxed{V = 8.370\,u^3}$

Student Response 2 (Score: 6)



Work for problem 1(a)

$\int_0^2 (\sin(\pi x) - x^3 + 4x)\,dx = 4$

Work for problem 1(b)

$x^3 - 4x = -2$ $x = .539$ or $x = 1.675$

$\int_{.539}^{1.675} (-2 - x^3 + 4x)\,dx$

Work for problem 1(c)

[illegible] $V = 2\pi\int_0^2 (r\,h)\,dx$

$r =$

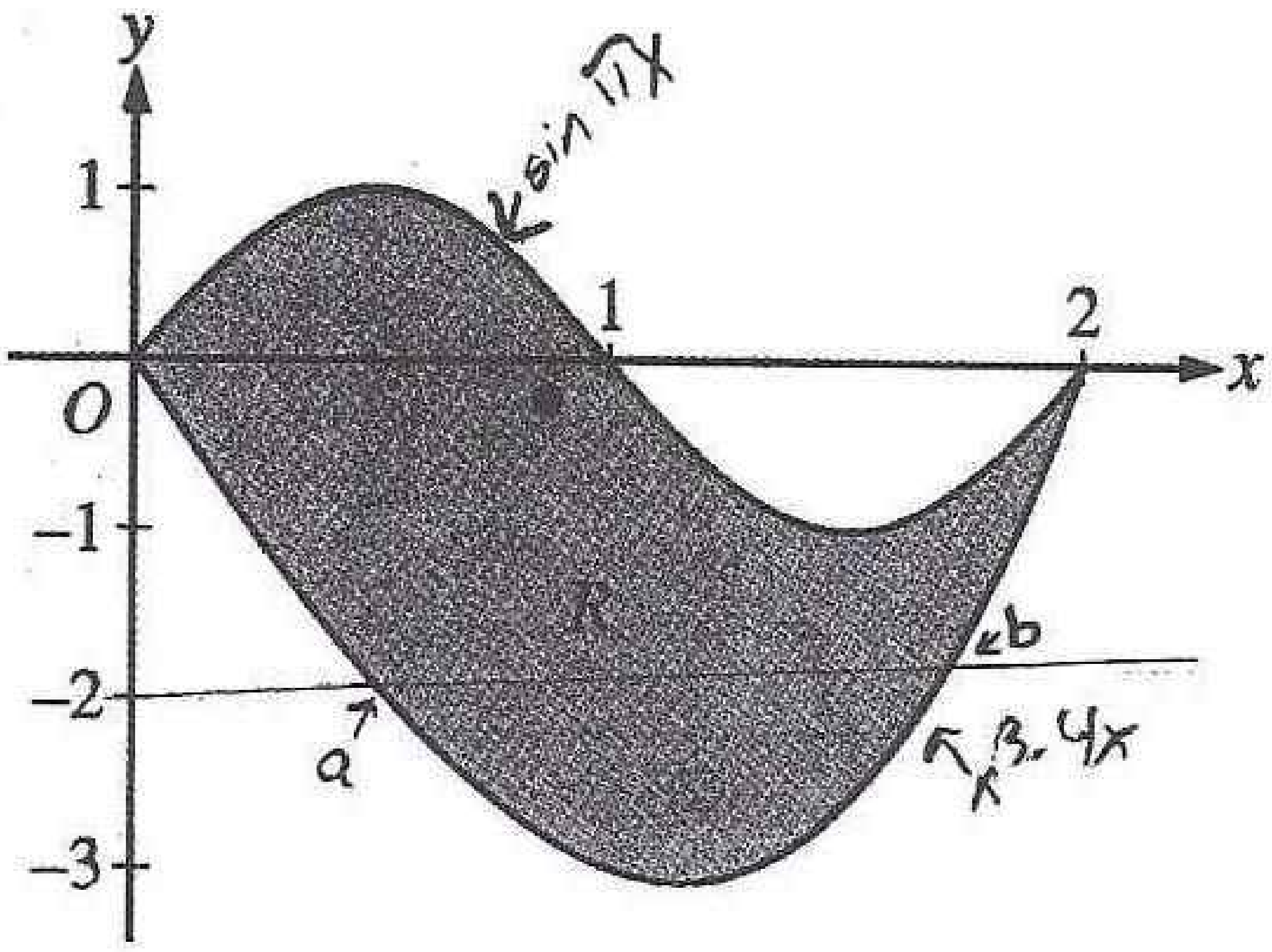
$h =$

Work for problem 1(d)

$V = 2\pi\int_0^2 ((\sin 6x) - x^3 + 4x)(3 - x))\,dx$

≈ 52.590

Student Response 3 (Score: 4)



Work for problem 1(a)

$$R=\int_0^2 \sin\pi x-(x^3-4x)\,dx$$

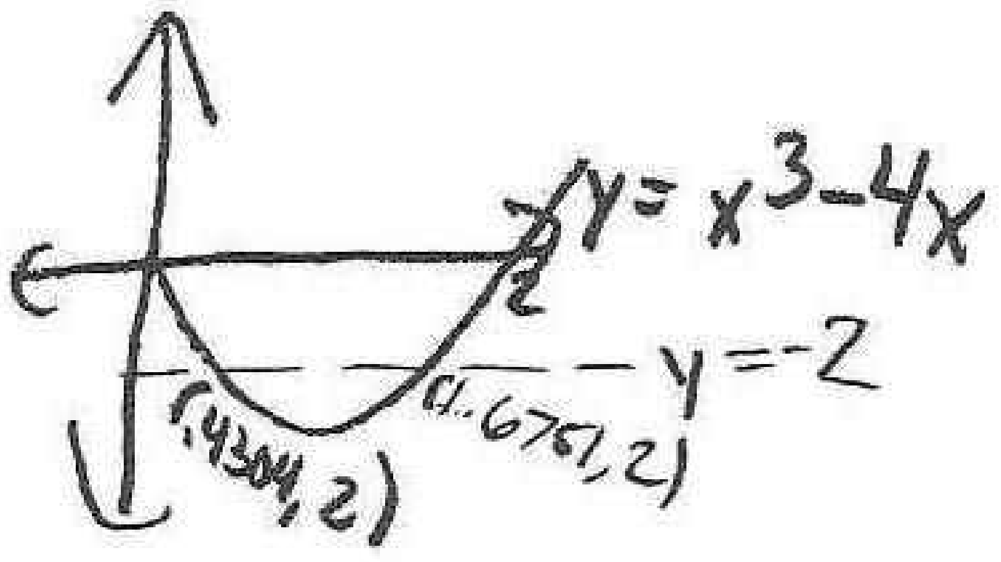
$$=\frac{-\cos(\pi x)}{\pi}-\frac{x^4}{4}+2x^2\Big]_0^2$$

$$=4$$

Work for problem 1(b)

$a=.4304$ $b=1.6751$

$$A=\int_a^b (-2)-(x^3-4x)\,dx$$



Student Response 3 *(continued)*

Work for problem 1(c)

$$V = \pi \int_0^2 \frac{\sqrt{3}}{4}(\sin \pi x - (x^3 - 4x))\,dx$$

$$= 4.321\pi \text{ or } 13.574$$

Work for problem 1(d)

$h(x) = 3 - x$

Calculus AB/BC Question 2

Overview

This problem presented students with a table of data indicating the number of people $L(t)$ in line at a concert hall ticket office, sampled at seven times t during the 9 hours that tickets were being sold. (The question stated that $L(t)$ was twice differentiable.) Part (a) asked for an estimate for the rate of change of the number of people in line at a time that fell between the times sampled in the table. Students were to use data from the table to calculate an average rate of change to approximate this value. Part (b) asked for an estimate of the average number of people waiting in line during the first 4 hours and specified the use of a trapezoidal sum. Students needed to recognize that the computation of an average value involves a definite integral, approximate this integral with a trapezoidal sum, and then divide this total accumulation of people hours by 4 hours to obtain the average. Part (c) asked for the minimum number of solutions guaranteed for $L'(t) = 0$ during the 9 hours. Students were expected to recognize that a change in direction (increasing/decreasing) for a twice-differentiable function forces a value of 0 for its derivative. Part (d) provided the function $r(t) = 550te^{-t/2}$ tickets sold per hour as a model of the rate at which tickets were sold during the 9 hours and asked students to find the number of tickets sold in the first 3 hours, to the nearest whole number, using this model. Students needed to recognize that total tickets sold could be determined by a definite integral of the rate $r(t)$ at which tickets were sold.

The mean score for this question was 3.36 for AB students and 5.10 for BC students. In parts (a) and (b), a graphing calculator can be used to compute final answers. Students who do this work by hand must have a correct arithmetic expression for their answer, with values correctly pulled from the table, to earn the answer points. In part (a) students should use the interval $[4, 7]$ to produce their estimate of the derivative at $t = 5.5$. In part (b) students must use a trapezoidal sum, not the Trapezoidal Rule, since Δt is not constant for data in the table. It is important that when given tabular data, students avoid making assumptions about the behavior of the function between data points. Since L is assumed to be differentiable, it is continuous, and we can draw conclusions using the Intermediate Value Theorem on each of the intervals in the table. However, we do not have information about the increasing or decreasing behavior of L on any subinterval. Students need practice with writing justifications based on tabular data. In part (d) students can and should evaluate the integral by using the graphing calculator rather than antidifferentiating, a task that would involve integration by parts.

Commentary on Student Responses

Student Response 1 (Score: 9)

The student earned all 9 points. In part (c) a more complete justification for the existence of the three relative extreme points is desirable, but the student's response earned all 3 points.

Student Response 2 (Score: 6)

The student earned 6 points: 2 points in part (a), 1 point in part (b), 2 points in part (c), and 1 point in part (d). In part (a) the student has a correct estimate (resulting from a correct difference quotient) and correct units. In part (b) the trapezoidal sum is correct and earned the first point. The missing bracket was ignored. The student did not earn the second point due to the arithmetic error. In part (c) the student earned the first point by considering "The number of people waiting to buy tickets" changing from increasing to decreasing, etc. The student did not earn the second point as a result of claiming that $L(t)$ is increasing on the interval $[0, 3]$, which need not be true. The student earned the third point with the correct answer of 3. In part (d) the student earned the first point for a correct integrand but did not earn the second point due to an incorrect answer. The student's work shows confusion between the functions r and L.

Student Response 3 (Score: 3)

The student earned 3 points: no points in part (a), no points in part (b), 1 point in part (c), and 2 points in part (d). In part (b) the student's expression ignores the fact that the second subinterval of $[0, 4]$ has length 2. In part (c) the student has the correct answer and earned the third point. The student did not earn the first 2 points for stating that $L(t)$, rather than $L'(t)$, changes from positive to negative or vice versa three times. Even if the student had referred to the correct function $L'(t)$, the second point requires more justification. (The statement that "the graph of $L'(t)$ to cross the x-axis" would, by itself, have earned the first point, since the "x" would have been read as "t.") In part (d) the student earned both points since the integrand is correct, and the given decimal value is accurate to the number of digits displayed (2). The final answer is one of the two acceptable integer answers (972 and 973). Moreover, the limits on the integral are correct.

Scoring Guidelines for Calculus AB/BC Question 2

t (hours)	0	1	3	4	7	8	9
$L(t)$ (people)	120	156	176	126	150	80	0

Concert tickets went on sale at noon $(t = 0)$ and were sold out within 9 hours. The number of people waiting in line to purchase tickets at time t is modeled by a twice-differentiable function L for $0 \le t \le 9$. Values of $L(t)$ at various times t are shown in the table above.

(a) Use the data in the table to estimate the rate at which the number of people waiting in line was changing at 5:30 P.M. $(t = 5.5)$. Show the computations that lead to your answer. Indicate units of measure.

(b) Use a trapezoidal sum with three subintervals to estimate the average number of people waiting in line during the first 4 hours that tickets were on sale.

(c) For $0 \le t \le 9$, what is the fewest number of times at which $L'(t)$ must equal 0 ? Give a reason for your answer.

(d) The rate at which tickets were sold for $0 \le t \le 9$ is modeled by $r(t) = 550te^{-t/2}$ tickets per hour. Based on the model, how many tickets were sold by 3 P.M. $(t = 3)$, to the nearest whole number?

(a) $L'(5.5) \approx \dfrac{L(7) - L(4)}{7 - 4} = \dfrac{150 - 126}{3} = 8$ people per hour

$$2 : \begin{cases} 1 : \text{estimate} \\ 1 : \text{units} \end{cases}$$

(b) The average number of people waiting in line during the first 4 hours is approximately

$$\frac{1}{4}\left(\frac{L(0) + L(1)}{2}(1 - 0) + \frac{L(1) + L(3)}{2}(3 - 1) + \frac{L(3) + L(4)}{2}(4 - 3)\right)$$

$= 155.25$ people

$$2 : \begin{cases} 1 : \text{trapezoidal sum} \\ 1 : \text{answer} \end{cases}$$

(c) L is differentiable on $[0, 9]$ so the Mean Value Theorem implies $L'(t) > 0$ for some t in $(1, 3)$ and some t in $(4, 7)$. Similarly, $L'(t) < 0$ for some t in $(3, 4)$ and some t in $(7, 8)$. Then, since L' is continuous on $[0, 9]$, the Intermediate Value Theorem implies that $L'(t) = 0$ for at least three values of t in $[0, 9]$.

$$3 : \begin{cases} 1 : \text{considers change in sign of } L' \\ 1 : \text{analysis} \\ 1 : \text{conclusion} \end{cases}$$

OR

The continuity of L on $[1, 4]$ implies that L attains a maximum value there. Since $L(3) > L(1)$ and $L(3) > L(4)$, this maximum occurs on $(1, 4)$. Similarly, L attains a minimum on $(3, 7)$ and a maximum on $(4, 8)$. L is differentiable, so $L'(t) = 0$ at each relative extreme point on $(0, 9)$. Therefore $L'(t) = 0$ for at least three values of t in $[0, 9]$.

OR

$$3 : \begin{cases} 1 : \text{considers relative extrema of } L \text{ on } (0, 9) \\ 1 : \text{analysis} \\ 1 : \text{conclusion} \end{cases}$$

[*Note: There is a function* L *that satisfies the given conditions with* $L'(t) = 0$ *for exactly three values of* t.]

(d) $\int_0^3 r(t)\,dt = 972.784$

There were approximately 973 tickets sold by 3 P.M.

$$2 : \begin{cases} 1 : \text{integrand} \\ 1 : \text{limits and answer} \end{cases}$$

Sample Student Responses for Calculus AB/BC Question 2

Student Response 1 (Score: 9)

t (hours)	0	1	3	4	7	8	9
$L(t)$ (people)	120	156	176	126	150	80	0

Work for problem 2(a)

rate at which L is changing at 5:30 is equal to $L'(5.5)$

$$L'(5.5) \approx \frac{L(7)-L(4)}{7-4} = \frac{150-126}{3} = 8 \frac{\text{people}}{\text{hour}}$$

Work for problem 2(b)

$$\text{avg value} = \frac{1}{b-a}\int_a^b L(t)\,dt$$

$$\int_0^4 L(t)\,dt \approx 1\left(\frac{120+156}{2}\right) + 2\left(\frac{156+176}{2}\right) + 1\left(\frac{176+126}{2}\right) = 621$$

so $$\frac{1}{4-0}\int_0^4 L(t)\,dt \approx \frac{621}{4} = 155.25 \text{ people}$$

Work for problem 2(c)

$L'(t)=0$ when $L(t)$ is at rel. min or rel. max.

$L(t)$ must have rel. max on $(1,4)$ and $(4,8)$ and must have rel. min on $(3,7)$. Therefore $L'(t)$ must equal zero at least 3 times on $[0,9]$

Work for problem 2(d)

total tickets sold $= \int_0^3 r(t)dt = 973$ tickets

Student Response 2 (Score: 6)

t (hours)	0	1	3	4	7	8	9
$L(t)$ (people)	120	156	176	126	150	80	0

Work for problem 2(a)

$$\frac{dL}{dt} = \frac{L(7) - L(4)}{7 - 4} = \frac{150-126}{3}$$

$$\frac{dL}{dt} = \boxed{8 \frac{\text{people}}{\text{hour}}}$$

Work for problem 2(b)

$$\int_0^4 L(t)\,dt \approx \frac{1}{4}\Big[1\left(\frac{156+120}{2}\right) + 2\left(\frac{176+156}{2}\right) + 1\left(\frac{176+126}{2}\right)$$

$$\frac{1}{4}(138 + 166 + 151) = \boxed{113.75 \text{ people}}$$

Student Response 2 *(continued)*

Work for problem 2(c)

For $0 \le t \le 9$, the least number of times that $L'(t)$ must equal 0 is 3. The number of people waiting to buy tickets increases from $0 \le t < 3$, decreases $3 < t < 4$, increases $4 < t < 7$, and decreases $7 < t \le 9$. Therefore $L'(t)$ switches from negative to positive once and positive to negative twice, requiring 3 times that $L'(t)$ must equal 0, by the Intermediate Value theorem.

Work for problem 2(d)

$$L(3) - L(0) = \int_0^3 550te^{-t/2}\,dt$$

$$L(3) - 120 = 972.7841192$$

$$L(3) = 1092.784 \approx 1093$$

$$L(3) \approx 1093 \text{ tickets}$$

Student Response 3 (Score: 3)

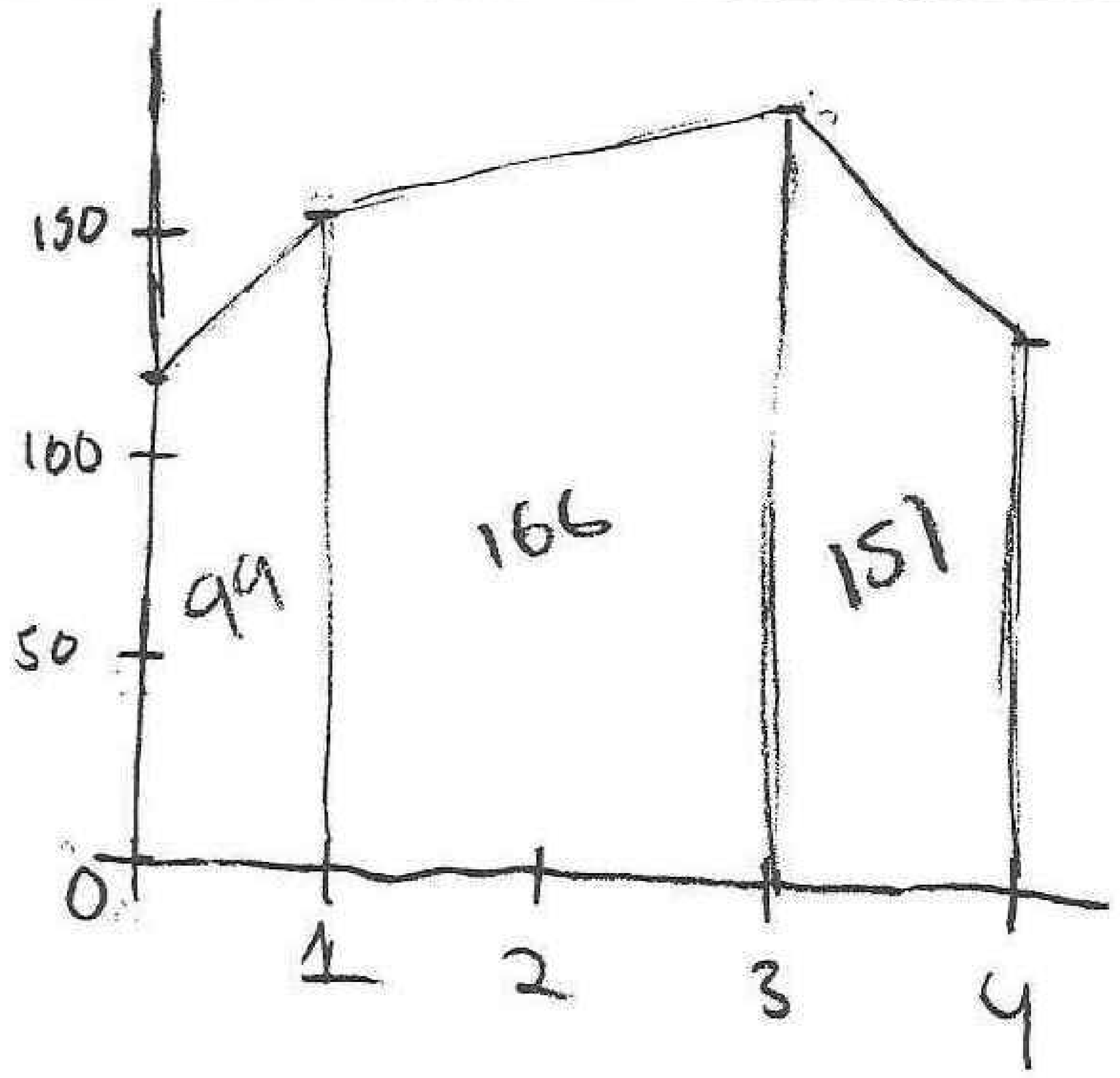
t (hours)	0	1	3	4	7	8	9
$L(t)$ (people)	120	156	176	126	150	80	0

Work for problem 2(a)

$4+7=\frac{11}{2}=5.5$

$126+150=$

Work for problem 2(b)



$$\frac{120+156}{2}+\frac{156+176}{2}+\frac{176+126}{2}=\frac{416}{4}$$

$$=104\ \frac{\text{people}}{\text{hour}}$$

Student Response 3 *(continued)*

Work for problem 2(c)

The fewest # of time $L'(t)$ must equal 0 is three, The $L(t)$ changes either from + to - or - to + 3 times which would cause the graph of $L'(t)$ to cross the x-axis 3 times.

Work for problem 2(d)

$$\int_6^3 550te^{-t/2}\,dt$$

$= 972.78$ Round up

$\boxed{973 \text{ Tickets}}$

Calculus AB Question 3

Overview

This problem presented students with a scenario in which oil leaking from a pipeline into a lake organizes itself as a dynamic cylinder whose height and radius change with time. The rate at which oil is leaking into the lake was given as 2000 cubic centimeters per minute. Part (a) was a related-rates problem; students needed to use the chain rule to differentiate volume, $V = \pi r^2 h$, with respect to time and determine the rate of change of the oil slick's height at an instant when the oil slick has radius 100 cm and height 0.5 cm, and its radius is increasing at 2.5 cm/min. In part (b) an oil recovery device arrives on the scene; as the pipeline continues to leak at 2000 cubic centimeters per minute, the device removes oil at the rate of $R(t) = 400\sqrt{t}$ cubic centimeters per minute, with t measured in minutes from the time the device began removing oil. Students were asked for the time t when the volume of the oil cylinder is greatest. They needed to recognize the rate of change of the volume of oil in the lake, $\frac{dV}{dt}$, as the difference between the rate at which oil enters the lake from the leak and the rate at which it is removed by the device. A sign analysis of $\frac{dV}{dt}$ or an application of the Second Derivative Test and the critical point theorem could justify that the critical point found yields a maximum value for the volume of the oil cylinder. Part (c) tested students' ability to use the Fundamental Theorem of Calculus to find the amount of oil in the lake at the time found in part (b), given that 60,000 cubic centimeters had already leaked when the recovery device began its task.

The mean score for this question was 2.45. Although a calculator could be used to assist in computing the final answer in part (a), its use was not needed. Students doing these calculations by hand need not simplify their numerical answer, but they must have a correct explicit arithmetic expression for $\frac{dh}{dt}$ to earn the answer point. In part (b) students must use a global argument to show that at time $t = 25$, the oil slick reaches its maximum volume. An argument such as stating that the second derivative is negative at $t = 25$ is only enough to show that at this time the oil slick reaches a relative maximum. In part (c) students earned points for a correct expression. Students were not penalized for continuing to evaluate the expression since this was not asked for in the question. As in all problems on the free-response section of the exam, students must show the work that leads to their answers to earn credit.

Commentary on Student Responses

Student Response 1 (Score: 9)

The student earned all 9 points.

Student Response 2 (Score: 6)

The student earned 6 points: 3 points in part (a), 2 points in part (b), and 1 point in part (c). In part (a) the student gives the correct symbolic expression for $\frac{dV}{dt}$ and earned the 2 derivative points. The student correctly substitutes 2000 for $\frac{dV}{dt}$ and 2.5 for $\frac{dr}{dt}$ in this expression, which earned the first point. The student has an error in the evaluation of the expression and did not earn the answer point. In part (b) the student earned the first 2 points for solving $2000 - 400\sqrt{t} = 0$ to find $t = 25$ as the time when the volume reaches a maximum. The justification given is only for a relative maximum, but a justification for an absolute maximum is required, so the third point was not earned. In part (c) the upper limit of integration is not correct, and so the student did not earn the limits and initial condition point. The student earned the integrand point.

Student Response 3 (Score: 4)

The student earned 4 points: 1 point in part (a), 1 point in part (b), and 2 points in part (c). In part (a) the student correctly notes that $\frac{dV}{dt} = 2000$ and that $\frac{dr}{dt} = 2.5$, earning the first point. The student does not use the product rule correctly. The student did not earn either of the derivative points and was not eligible for the answer point. In part (b) the student does not solve $2000 - 400\sqrt{t} = 0$ correctly. Only the first point was earned. The student does not provide any justification. Both points were earned in part (c). The initial condition is correct, and the student is allowed to import an incorrect value of t found in part (b) as the upper limit of integration. The integrand is correct.

Scoring Guidelines for Calculus AB Question 3

Oil is leaking from a pipeline on the surface of a lake and forms an oil slick whose volume increases at a constant rate of 2000 cubic centimeters per minute. The oil slick takes the form of a right circular cylinder with both its radius and height changing with time. (Note: The volume V of a right circular cylinder with radius r and height h is given by $V = \pi r^2 h$.)

(a) At the instant when the radius of the oil slick is 100 centimeters and the height is 0.5 centimeter, the radius is increasing at the rate of 2.5 centimeters per minute. At this instant, what is the rate of change of the height of the oil slick with respect to time, in centimeters per minute?

(b) A recovery device arrives on the scene and begins removing oil. The rate at which oil is removed is $R(t) = 400\sqrt{t}$ cubic centimeters per minute, where t is the time in minutes since the device began working. Oil continues to leak at the rate of 2000 cubic centimeters per minute. Find the time t when the oil slick reaches its maximum volume. Justify your answer.

(c) By the time the recovery device began removing oil, 60,000 cubic centimeters of oil had already leaked. Write, but do not evaluate, an expression involving an integral that gives the volume of oil at the time found in part (b).

(a) When $r = 100$ cm and $h = 0.5$ cm, $\frac{dV}{dt} = 2000\ \text{cm}^3/\text{min}$ and $\frac{dr}{dt} = 2.5\ \text{cm}/\text{min}$.

$$\frac{dV}{dt} = 2\pi r \frac{dr}{dt} h + \pi r^2 \frac{dh}{dt}$$

$$2000 = 2\pi(100)(2.5)(0.5) + \pi(100)^2 \frac{dh}{dt}$$

$$\frac{dh}{dt} = 0.038 \text{ or } 0.039\ \text{cm}/\text{min}$$

$$4 : \begin{cases} 1 : \dfrac{dV}{dt} = 2000 \text{ and } \dfrac{dr}{dt} = 2.5 \\ 2 : \text{expression for } \dfrac{dV}{dt} \\ 1 : \text{answer} \end{cases}$$

(b) $\frac{dV}{dt} = 2000 - R(t)$, so $\frac{dV}{dt} = 0$ when $R(t) = 2000$.
This occurs when $t = 25$ minutes.
Since $\frac{dV}{dt} > 0$ for $0 < t < 25$ and $\frac{dV}{dt} < 0$ for $t > 25$, the oil slick reaches its maximum volume 25 minutes after the device begins working.

$$3 : \begin{cases} 1 : R(t) = 2000 \\ 1 : \text{answer} \\ 1 : \text{justification} \end{cases}$$

(c) The volume of oil, in cm^3, in the slick at time $t = 25$ minutes is given by $60{,}000 + \int_0^{25} (2000 - R(t))\, dt$.

$$2 : \begin{cases} 1 : \text{limits and initial condition} \\ 1 : \text{integrand} \end{cases}$$

Sample Student Responses for Calculus AB Question 3

Student Response 1 (Score: 9)

Work for problem 3(a)

$$\frac{dV}{dt} = 2000 \frac{cm^3}{min}$$

$$V = \pi r^2 h$$

$r = 100 cm$

$h = .5 cm$

$$\frac{dr}{dt} = 2.5 \frac{cm}{min}$$

$$\frac{dV}{dt} = \pi\left(r^2\frac{dh}{dt} + 2r\frac{dr}{dt}h\right)$$

$$\frac{dV}{dt} - 2\pi r\frac{dr}{dt}h = \pi r^2\frac{dh}{dt}$$

$$\frac{\frac{dV}{dt} - 2\pi r\frac{dr}{dt}h}{\pi r^2} = \frac{dh}{dt}$$

$$\frac{dh}{dt} = \frac{2000 - 2\pi(100)(2.5)(.5)}{\pi(100)^2}$$

$$\frac{dh}{dt} = .038 \text{ cm/min}$$

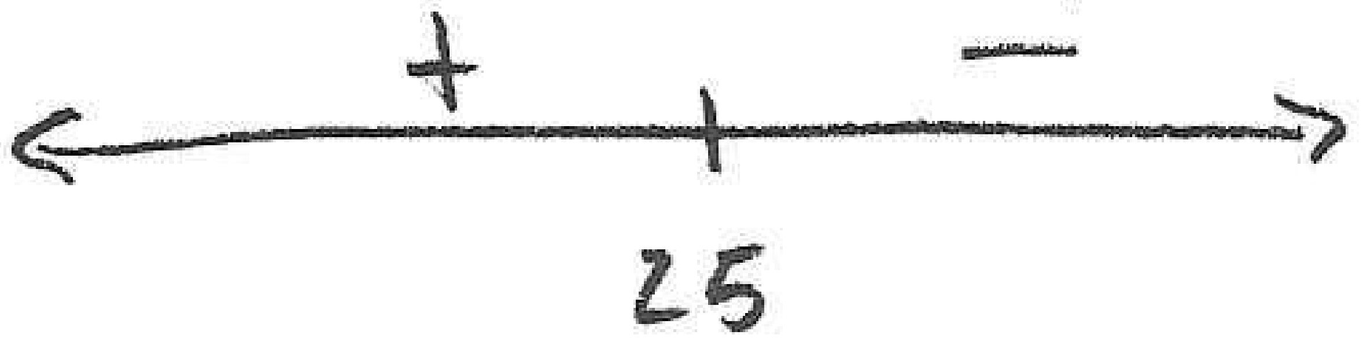
Work for problem 3(b)

In = 2000

Out = $400\sqrt{t}$

$$V'(t) = 2000 - 400\sqrt{t} = 0$$

$$t = 25$$



The oil slick reaches its maximum volume at $t = 25$ minutes b/c the rate at which oil enters and leaves at 25 minutes $V'(25) = 0$ and $V'(t) > 0$ for all $t < 25$ and $V'(t) < 0$ for all $t > 25$.

Work for problem 3(c)

$$V(t) = 60000 + \int_0^{25} (2000 - 400\sqrt{t})\,dt$$

Student Response 2 (Score: 6)

Work for problem 3(a)

$$V = \pi r^2 h$$

$$\frac{dV}{dt} = 2\pi r\, h \frac{dr}{dt} + \pi r^2 \cdot \frac{dh}{dt}$$

$$2000 = 2\pi . 100 . 0.5 . 2.5 + \pi . 100^2 . \frac{dh}{dt}$$

$$100^2 \pi \frac{dh}{dt} = 2000 - 125\pi \Rightarrow \frac{dh}{dt} = \frac{2000 - 125\pi}{100^2 \pi} \text{ cm/min}$$

Work for problem 3(b)

$A(t)$: volume of oil at time t

$A'(t) = V'(t) - R(t) = 2000 - 400\sqrt{t}$ cm³/min

$A'(t) = 0 \Rightarrow 2000 - 400\sqrt{t} = 0 \rightarrow \sqrt{t} = 5 \Rightarrow t = 25$

$A''(t) = -400 \cdot \frac{1}{2} \cdot t^{-1/2} = \frac{-200}{\sqrt{t}}$

$A''(25) = \frac{-200}{\sqrt{25}} = -40 < 0 \Rightarrow$ relative maximum at $t = 25$

Work for problem 3(c)

$A(5) = 60000 + \int_0^5 (2000 - 400\sqrt{t})\,dt$ (cm³)

Student Response 3 (Score: 4)

Work for problem 3(a)

$r = 100$ cm

$h = .5$ cm

$\frac{dv}{dt} = 2000 \frac{cm^3}{min}$

$\frac{dr}{dt} = 2.5$

$V = \pi R^2 h$

$\frac{dv}{dt} = \pi(2R)\frac{dR}{dt}\frac{dh}{dt}$

$2000 = \pi(200)(2.5)\frac{dh}{dt}$

$\frac{4}{\pi} = \frac{dh}{dt}$

$\frac{dh}{dt} = 1.273$ cm/min

Student Response 3 *(continued)*

Work for problem 3(b)

Let $X(t)=$ total cm^3 of oil entering pond

$$x(t) = \int 2000 - 400\sqrt{t}$$

$$x'(t) = 2000 - 400\sqrt{t} = 0$$

$$2000 = 400\sqrt{t}$$

$$\sqrt{t} = 4$$

$$t = 2 \text{ seconds}$$

Work for problem 3(c)

$$60{,}000 + \int_0^2 2000 - 400\sqrt{t}\, dt$$

Calculus AB/BC Question 4

Overview

This problem presented students with the graph of a velocity function for a particle in motion along the *x*-axis for $0 \le t \le 6$. Areas of regions between the velocity curve and the *t*-axis were also given. Part (a) asked for the time and position of the particle when it is farthest left, so students needed to know that velocity is the derivative of position, and they had to be able to determine positions at critical times from the particle's initial position and areas of regions bounded by the velocity curve and the *t*-axis. Part (b) tested knowledge of the Intermediate Value Theorem applied to information about the particle's position function derived from its initial position and the supplied graph of its derivative. Part (c) asked students to interpret information about the speed of the particle from the velocity graph: namely, that if velocity is negative but increasing, then its absolute value, speed, is decreasing. Part (d) asked for the time intervals over which acceleration is negative, so students had to recognize that acceleration is the derivative of velocity. The sign of acceleration can be read from the intervals of increase/decrease of the velocity function.

The mean score for this question was 2.60 for AB students and 4.26 for BC students. No calculator was allowed for the question, but calculations were straightforward. The problem did require several justifications. It is best if students use complete sentences when communicating their justifications. In part (a) students needed to check the position at both $t = 3$ and $t = 6$ to assure that the leftmost position occurs at time $t = 3$. If a student does not check position at time $t = 6$, the student can only conclude that the position at $t = 3$ is the farthest left for times near $t = 3$. In part (b) using the fact that position is continuous and noting the positions at $t = 0$, $t = 3$, $t = 5$, and $t = 6$, students can conclude there are at least three times at which the particle is located at position $x = 8$. To conclude there are exactly three times, students need to indicate that the particle's motion is monotonic on each of the intervals $[0, 3]$, $[3, 5]$, and $[5, 6]$. In part (c) students need to have included correct reasoning in order to have earned the point. In part (d) the reasoning or justification was a separate point. Students earned a point for the correct answer in this part, even without correct justification.

Commentary on Student Responses

Student Response 1 (Score: 9)

The student earned all 9 points.

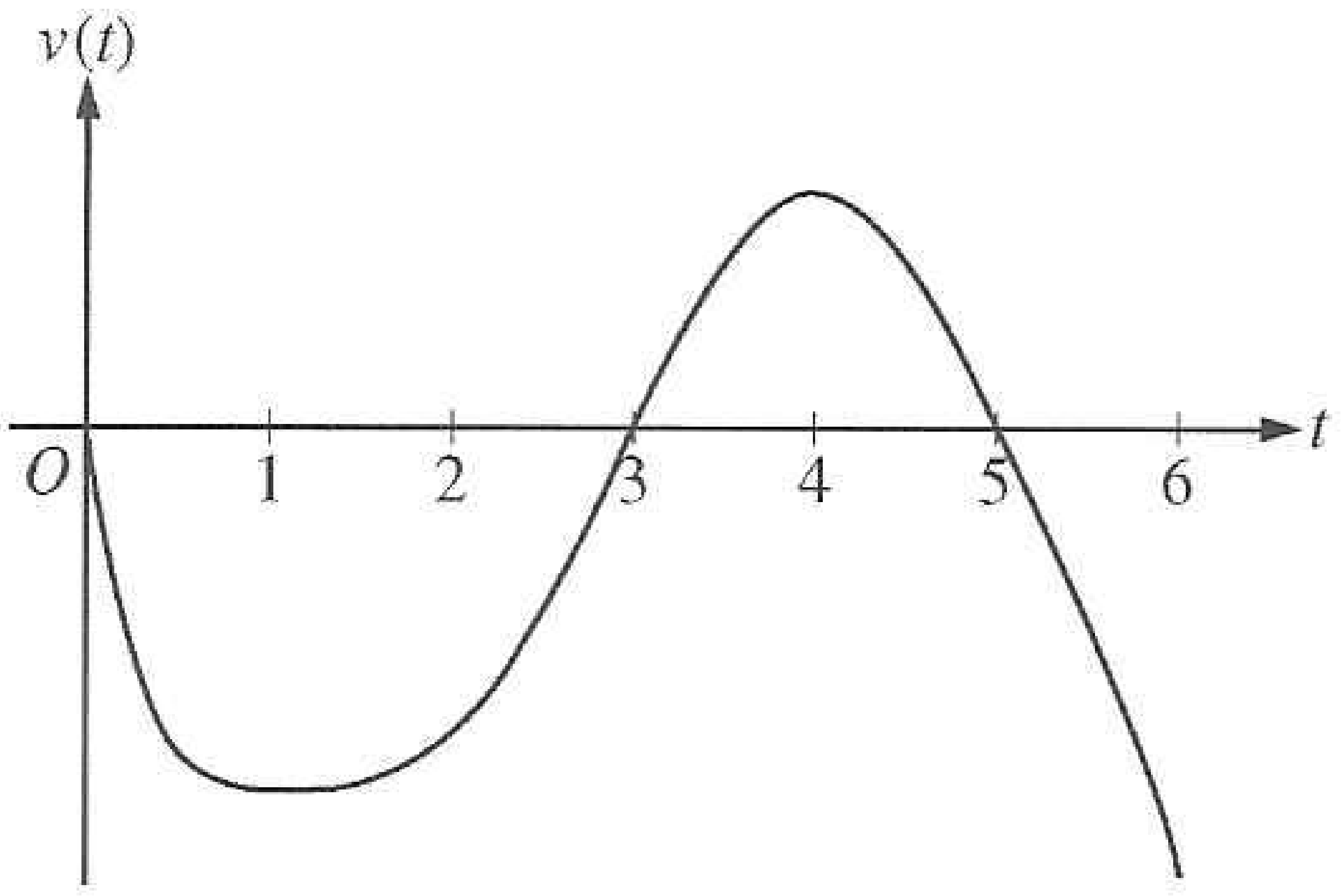
Student Response 2 (Score: 6)

The student earned 6 points: 1 point in part (a), 3 points in part (b), no points in part (c), and 2 points in part (d). In part (a) the student earned the first point for considering $t = 3$. The student does not present a complete justification that considers the entire time interval, so the second and third points were not earned. In part (b) the student earned all 3 points. In part (c) the student incorrectly concludes that speed is increasing and did not earn the point. In part (d) the student earned both points. The statements that "acceleration is negative" and the "slope of $v(t)$ gives $a(t)$" were sufficient to earn the justification point.

Student Response 3 (Score: 4)

The student earned 4 points: 1 point in part (a), no points in part (b), 1 point in part (c), and 2 points in part (d). In part (a) the student earned the first point, but no justification is provided. The student did not earn the second or third points. In part (b) the student does not consider the particle's movement over the time interval from $t = 5$ to $t = 6$, which leads to an incorrect conclusion. In part (c) the student draws a reasonable, labeled speed graph and earned the point for reaching the correct conclusion. The student earned both points in part (d).

Scoring Guidelines for Calculus AB/BC Question 4



Graph of v

A particle moves along the x-axis so that its velocity at time t, for $0 \le t \le 6$, is given by a differentiable function v whose graph is shown above. The velocity is 0 at $t = 0$, $t = 3$, and $t = 5$, and the graph has horizontal tangents at $t = 1$ and $t = 4$. The areas of the regions bounded by the t-axis and the graph of v on the intervals $[0, 3]$, $[3, 5]$, and $[5, 6]$ are 8, 3, and 2, respectively. At time $t = 0$, the particle is at $x = -2$.

(a) For $0 \le t \le 6$, find both the time and the position of the particle when the particle is farthest to the left. Justify your answer.

(b) For how many values of t, where $0 \le t \le 6$, is the particle at $x = -8$? Explain your reasoning.

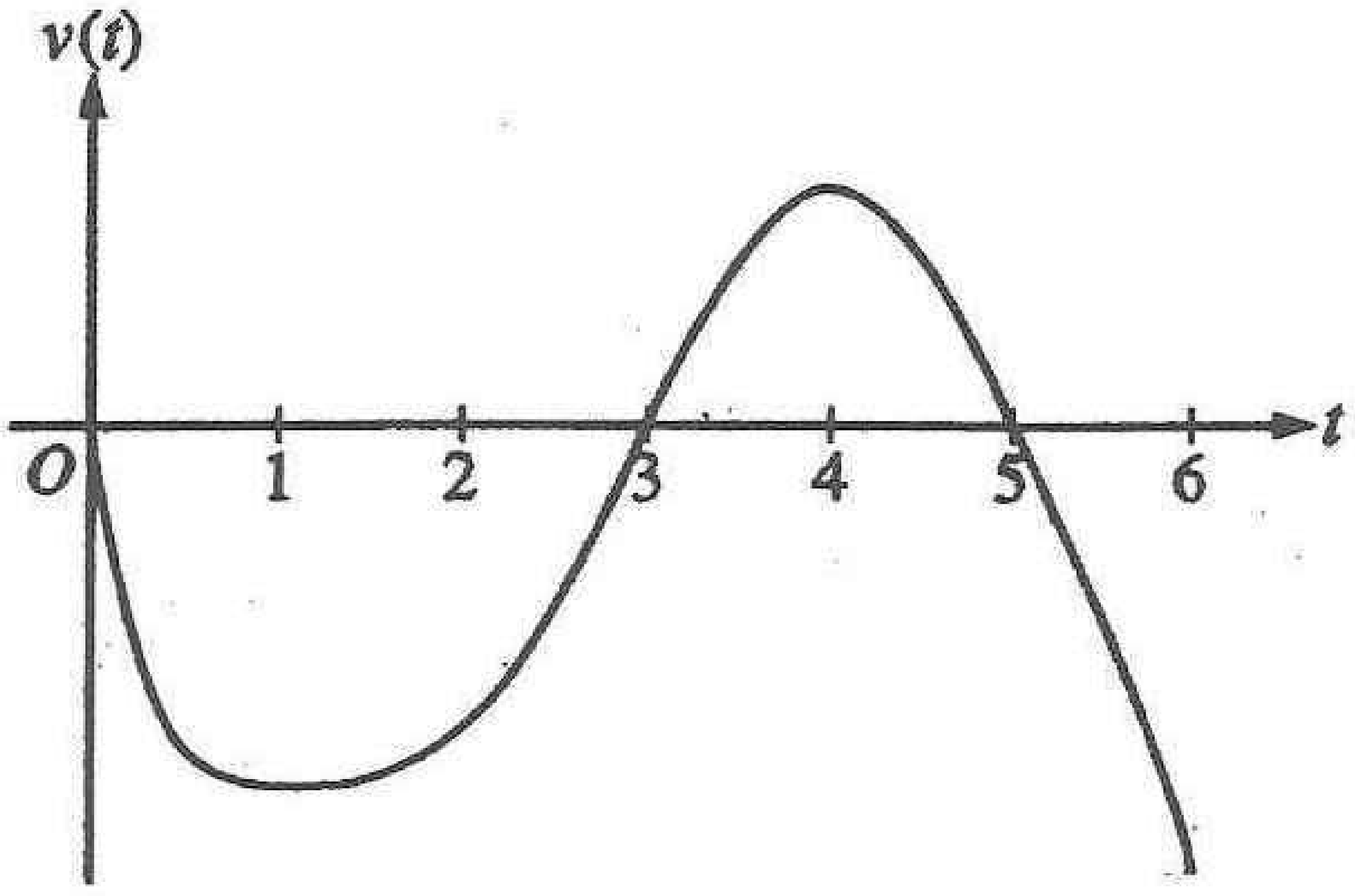
(c) On the interval $2 < t < 3$, is the speed of the particle increasing or decreasing? Give a reason for your answer.

(d) During what time intervals, if any, is the acceleration of the particle negative? Justify your answer.

(a) Since $v(t) < 0$ for $0 < t < 3$ and $5 < t < 6$, and $v(t) > 0$ for $3 < t < 5$, we consider $t = 3$ and $t = 6$. $x(3) = -2 + \int_0^3 v(t)\,dt = -2 - 8 = -10$ $x(6) = -2 + \int_0^6 v(t)\,dt = -2 - 8 + 3 - 2 = -9$ Therefore, the particle is farthest left at time $t = 3$ when its position is $x(3) = -10$.	$3 : \begin{cases} 1 : \text{identifies } t = 3 \text{ as a candidate} \\ 1 : \text{considers } \int_0^6 v(t)\,dt \\ 1 : \text{conclusion} \end{cases}$
(b) The particle moves continuously and monotonically from $x(0) = -2$ to $x(3) = -10$. Similarly, the particle moves continuously and monotonically from $x(3) = -10$ to $x(5) = -7$ and also from $x(5) = -7$ to $x(6) = -9$. By the Intermediate Value Theorem, there are three values of t for which the particle is at $x(t) = -8$.	$3 : \begin{cases} 1 : \text{positions at } t = 3,\ t = 5, \\ \quad \text{and } t = 6 \\ 1 : \text{description of motion} \\ 1 : \text{conclusion} \end{cases}$
(c) The speed is decreasing on the interval $2 < t < 3$ since on this interval $v < 0$ and v is increasing.	1 : answer with reason
(d) The acceleration is negative on the intervals $0 < t < 1$ and $4 < t < 6$ since velocity is decreasing on these intervals.	$2 : \begin{cases} 1 : \text{answer} \\ 1 : \text{justification} \end{cases}$

Sample Student Responses for Calculus AB/BC Question 4

Student Response 1 (Score: 9)



Graph of v

Work for problem 4(a)

Let $x(t)$ be the position of the particle. Note that $\int_a^b v(t)\,dt = x(b) - x(a)$. Thus $x(t_0) = x(0) + \int_0^{t_0} v(t)\,dt$. Therefore $x(t_0)$ is farthest to the left at whatever time $\int_0^{t_0} v(t)\,dt$ is most negative since $x(0)$ is constant. $\int_0^{t_0} v(t)\,dt$ is farthest left at either $t=3$ or $t=6$ since it is decreasing from 0 to 3, increasing from 3 to 5, and decreasing from 0 to 6. $\int_0^3 v(t)\,dt = -8$ and $\int_0^6 v(t)\,dt = -8 + 3 - 2 = -7$, as given. Therefore $\int_0^{t_0} v(t)\,dt$ is most negative at $t_0 = 3$, and $x(t_0)$ is farthest left at $\boxed{t_0 = 3.}$ $x(3) = x(0) + \int_0^3 v(t)\,dt = \boxed{-10}$

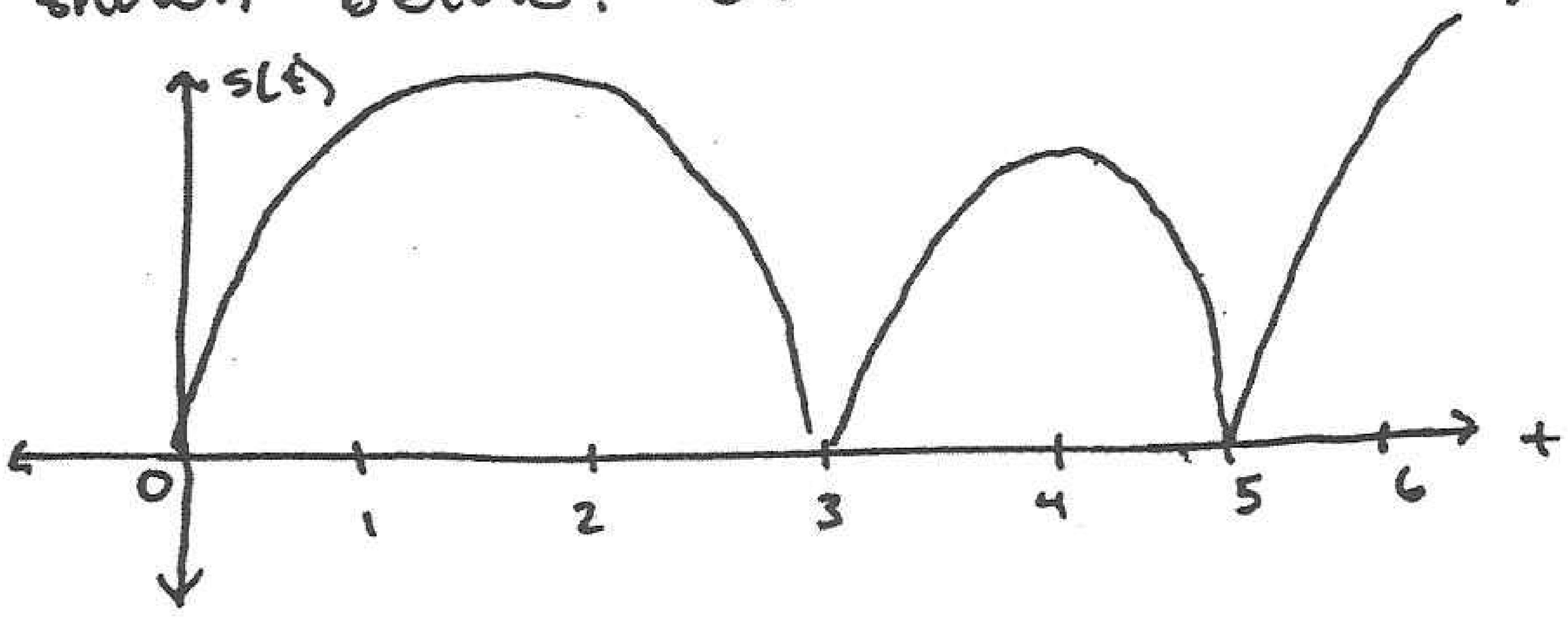
Work for problem 4(b)

Because $v(t)$ is continuous, the position function is continuous. Let $x(t)$ denote the position. Note that $x(t)$ is decreasing on the interval $0 < t < 3$ because $v(t)$ is negative on that interval. Also note that $x(0) = -2$ (given) and $x(3) = -10$ (see 4a). Because it's continuous and always decreasing for $0 < t < 3$, $x(t)$ must thus hit -8 exactly once between $t=0$ and $t=3$.

Similarly, $x(t)$ must hit -8 for $3 < t < 5$ exactly once since $x(3) = -10$, $x(5) = -7$, and $x(t)$ is always increasing on the interval $3 < t < 5$. By the same logic, $x(t)$ hits -8 exactly once on the interval $5 < t < 6$, for a total of $\boxed{3 \text{ times}}$ for $0 \le t \le 6$.

Work for problem 4(c)

Speed is the magnitude of velocity, or $|v(t)|$. Its graph is shown below: (speed is denoted $s(t)$)

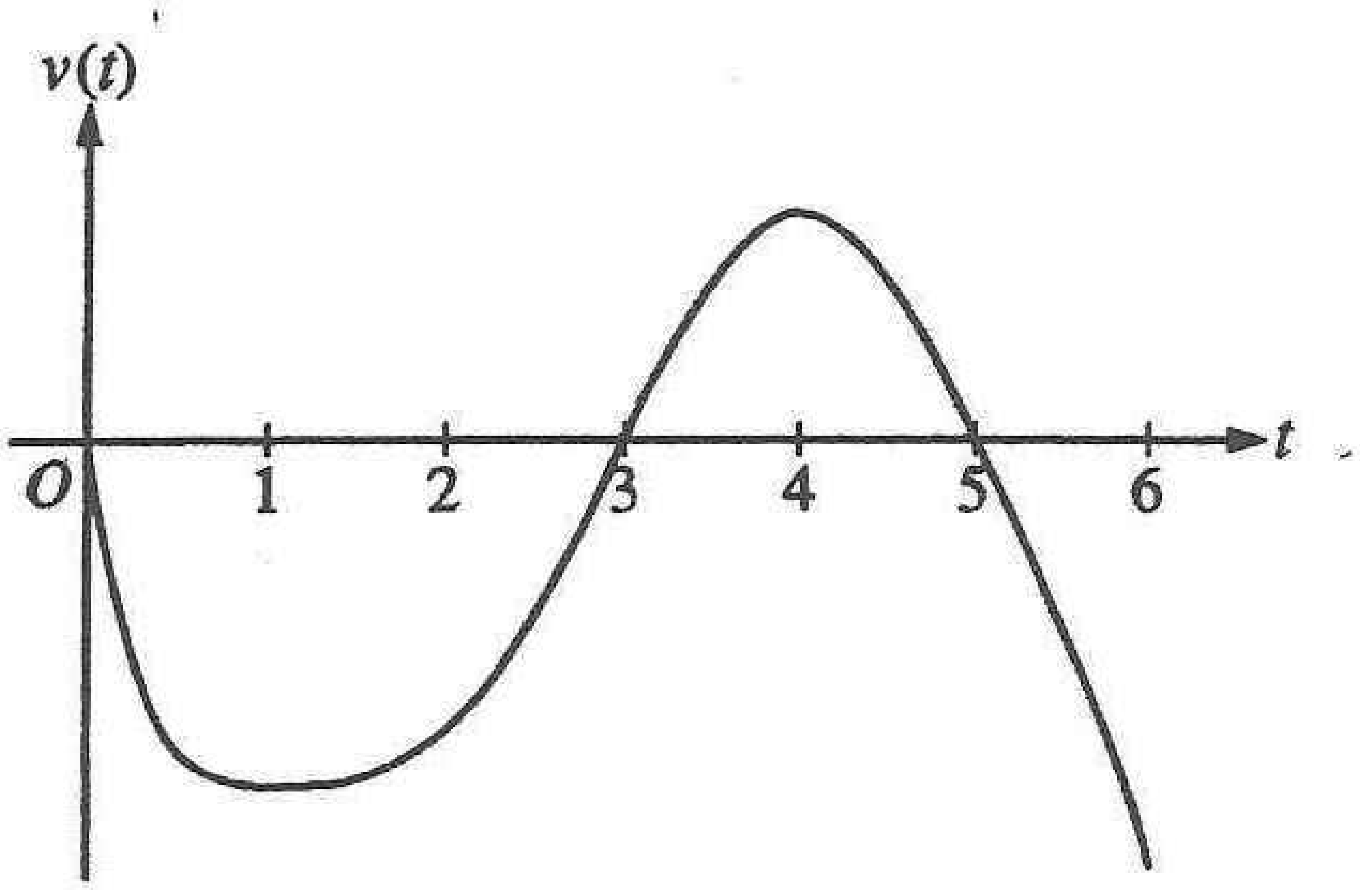


It is clear ~~that~~, since $v(t)$ is increasing on the interval $2<t<3$ ~~but~~ and since $s(t) = |v(t)| = -v(t)$ on the interval $2<t<3$, that $s(t)$ is decreasing on the interval $2<t<3$.

Work for problem 4(d)

Acceleration is the derivative of velocity, so acceleration is negative when the velocity is decreasing. It is known that the acceleration is 0 only at $t=1$ and $t=4$; also, it is obvious that $v(4) > v(0) > v(1) > v(6)$, so velocity is increasing on the interval $(1, 4)$ and decreasing on the interval $(0,1) \cup (4,6)$. Thus acceleration is negative on the intervals $0<t<1$ and $4<t<6$.

Student Response 2 (Score: 6)



Graph of v

[redacted]

Work for problem 4(a)

farthest left at $t=3$ because

$\int_0^3 f(x)$ yields -8 distance traveled

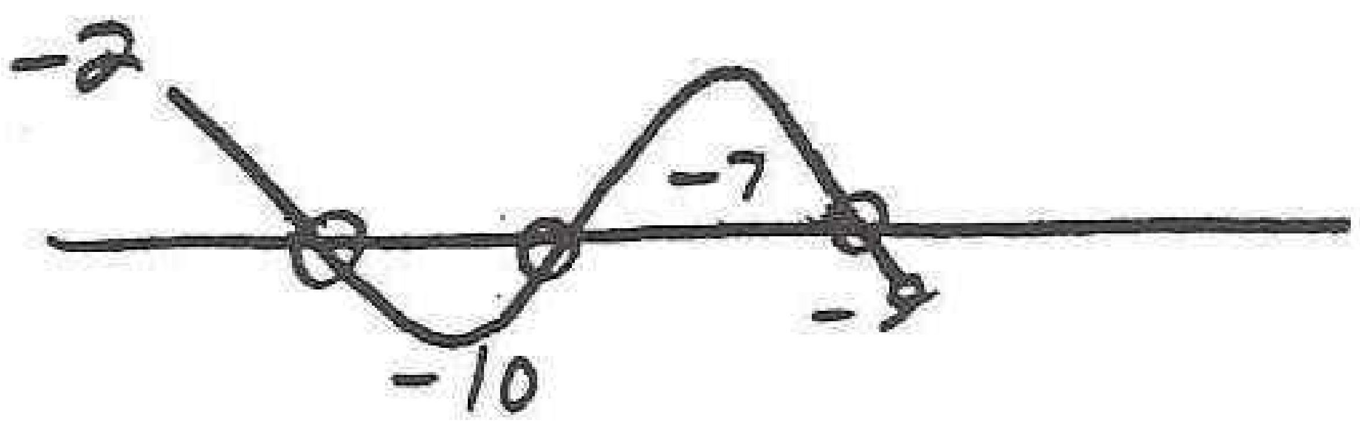
at $t=3$ $x=-10$

Work for problem 4(b)

if starts at -2 decreases to $x=-10$

at $t=3$ increases to $x=-7$ at $t=5$ then decreases agai

to $x=9$ at $x=-6$ particle is at $x=-8$



3 times

Work for problem 4(c)

~~magnitude of velocity vector = speed~~

increasing $\sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2} > 0$

Work for problem 4(d)

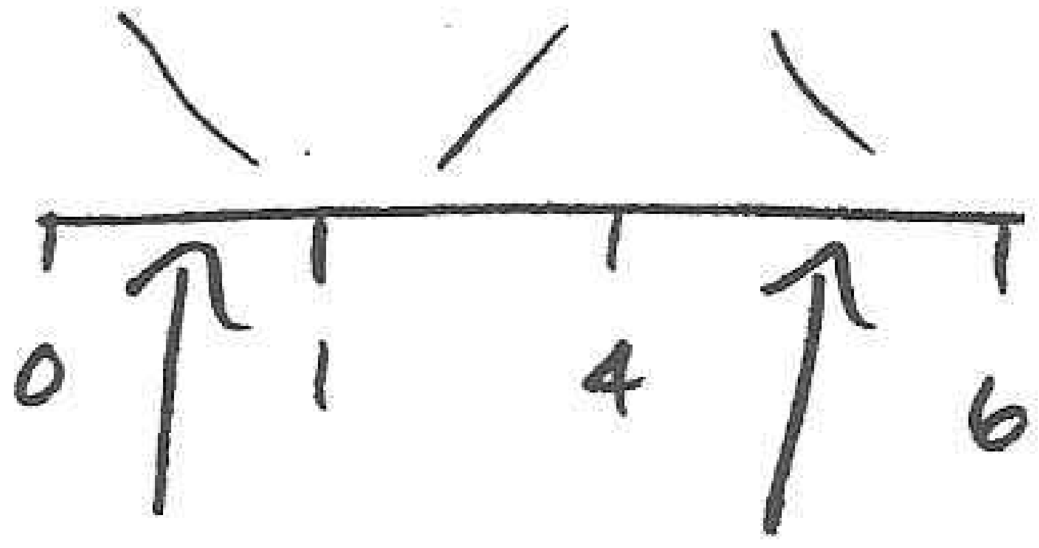
acceleration is negative for

$0 < t < 1$

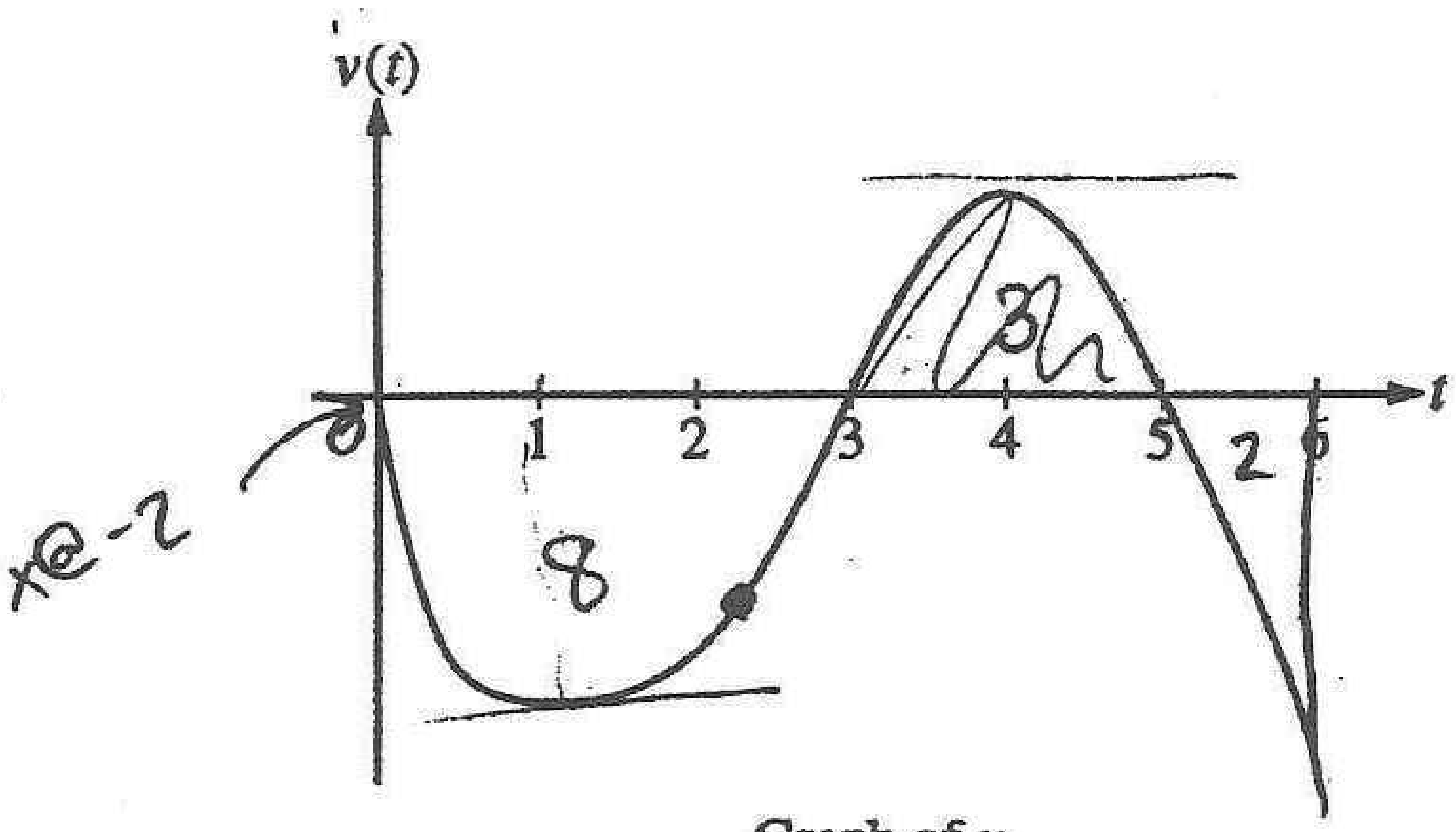
$4 < t < 6$

slope of $v(t)$ gives $a(t)$

critical values



Student Response 3 (Score: 4)



Graph of v

Work for problem 4(a)

time = $\boxed{3}$

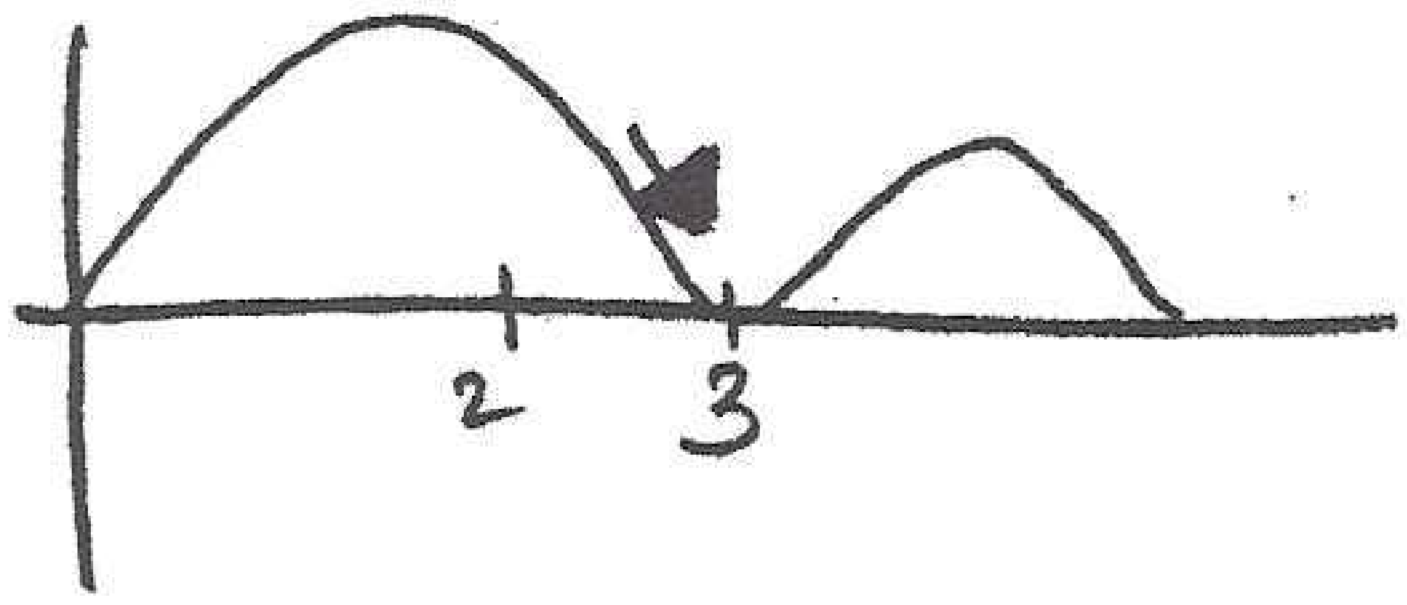
position = $-2-8=\boxed{-10}$

Work for problem 4(b)

The particle is at x=8 twice. By starting at a position of -2, it reaches -8 when t=6, then x goes to -10 and comes back up to -8 between t=3 and t=5.

Work for problem 4(c)

The speed is ~~increasing~~ decreasing because speed is the absolute value of v(t). From $2 < t < 3$, the speed is decreasing as shown in illustration below.



Work for problem 4(d)

The acceleration is negative when v(t) has a negative slope. Therefore, the acc. is negative from $0 \leq t 4$, and $4 < t \leq 6$.

Calculus AB Question 5

Overview

This problem presented students with a separable differential equation. In part (a) they were asked to sketch its slope field at nine sample points. Part (b) asked for the solution to the differential equation with a given initial condition. The solution involved selection of the portion of $\ln|y-1|$ that includes the initial condition. Part (c) asked for the limit of the solution from part (b) as $x \to \infty$.

The mean score for this question was 3.70. Calculations were not required to be shown in part (a). Students needed to sketch slope segments that were long enough to see that they were straight and that they had approximately the correct slope. Slope segments should not be drawn so large that they intersect. In such cases it appears that solutions to the given differential equation would intersect, something that does not happen with this differential equation. Students should avoid placing slope segments where $\frac{dy}{dx}$ is undefined. In this question this was along the y-axis. In part (b) when antidifferentiating, it is important that students include the absolute values around the argument of the natural logarithm. These absolute values can be removed once the appropriate solution that passes through the initial value is determined. To earn the answer point, students needed to arrive at an explicit expression for $f(x)$. In part (c) it was sufficient for a student to give the answer to earn the point.

Commentary on Student Responses

Student Response 1 (Score: 9)

The student earned all 9 points.

Student Response 2 (Score: 6)

The student earned 6 points: 1 point in part (a), 5 points in part (b), and no points in part (c). In part (a) the student earned the first point but has an incorrect slope at the point $(-1, 0)$. The student did not earn the second point. In part (b) the student has correct work until determining $y = f(x)$, at which point he or she does not choose the branch that contains the initial condition.

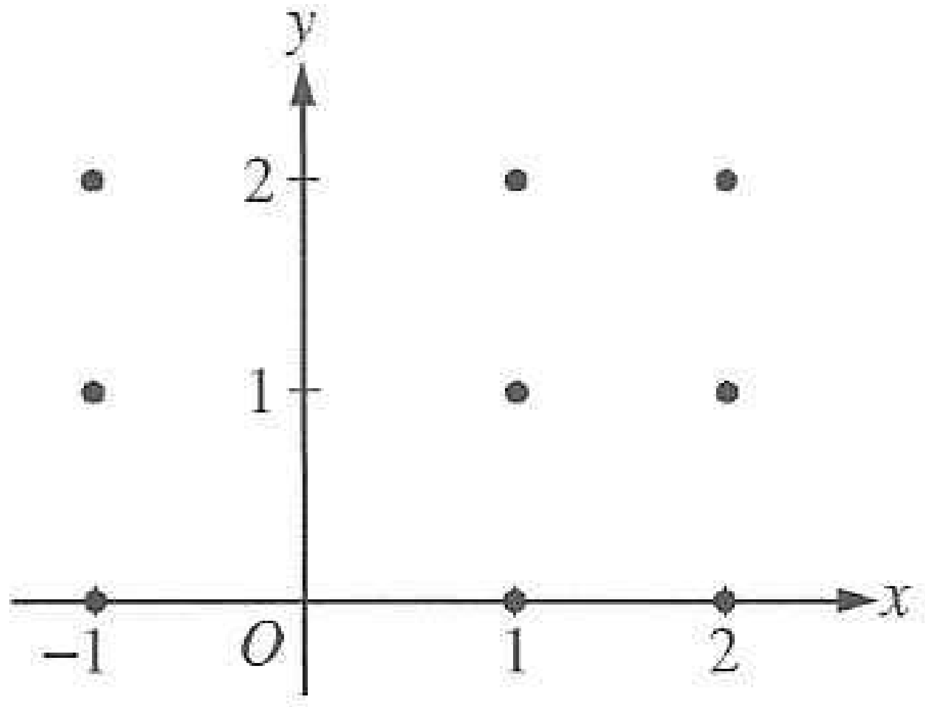
Student Response 3 (Score: 4)

The student earned 4 points: 1 point in part (a), 3 points in part (b), and no points in part (c). In part (a) the student earned the first point but has an incorrect slope at the point $(-1, 0)$. The student did not earn the second point. In part (b) the student earned points for the separation of variables, the constant of integration, and the use of the initial condition. In part (c) the student is not eligible for the limit point.

Scoring Guidelines for Calculus AB Question 5

Consider the differential equation $\frac{dy}{dx} = \frac{y-1}{x^2}$, where $x \neq 0$.

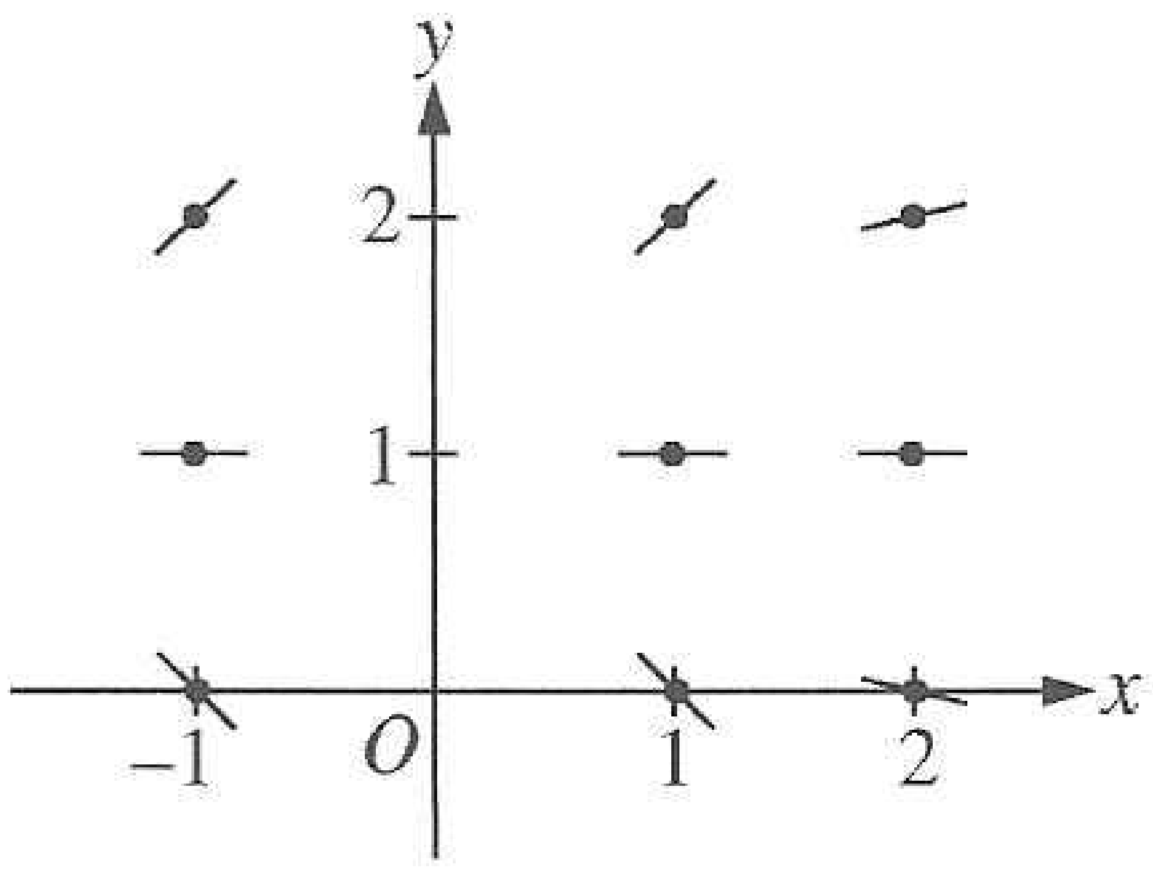
(a) On the axes provided, sketch a slope field for the given differential equation at the nine points indicated.
(Note: Use the axes provided in the exam booklet.)



(b) Find the particular solution $y = f(x)$ to the differential equation with the initial condition $f(2) = 0$.

(c) For the particular solution $y = f(x)$ described in part (b), find $\lim_{x\to\infty} f(x)$.

(a)



$$2 : \begin{cases} 1 : \text{zero slopes} \\ 1 : \text{all other slopes} \end{cases}$$

(b) $\frac{1}{y-1}\,dy = \frac{1}{x^2}\,dx$

$\ln|y-1| = -\frac{1}{x} + C$

$|y-1| = e^{-\frac{1}{x}+C}$

$|y-1| = e^{C}e^{-\frac{1}{x}}$

$y - 1 = ke^{-\frac{1}{x}}$, where $k = \pm e^{C}$

$-1 = ke^{-\frac{1}{2}}$

$k = -e^{\frac{1}{2}}$

$f(x) = 1 - e^{\left(\frac{1}{2}-\frac{1}{x}\right)}$, $x > 0$

$$6 : \begin{cases} 1 : \text{separates variables} \\ 2 : \text{antidifferentiates} \\ 1 : \text{includes constant of integration} \\ 1 : \text{uses initial condition} \\ 1 : \text{solves for } y \end{cases}$$

Note: max $3/6$ [1-2-0-0-0] if no constant of integration

Note: $0/6$ if no separation of variables

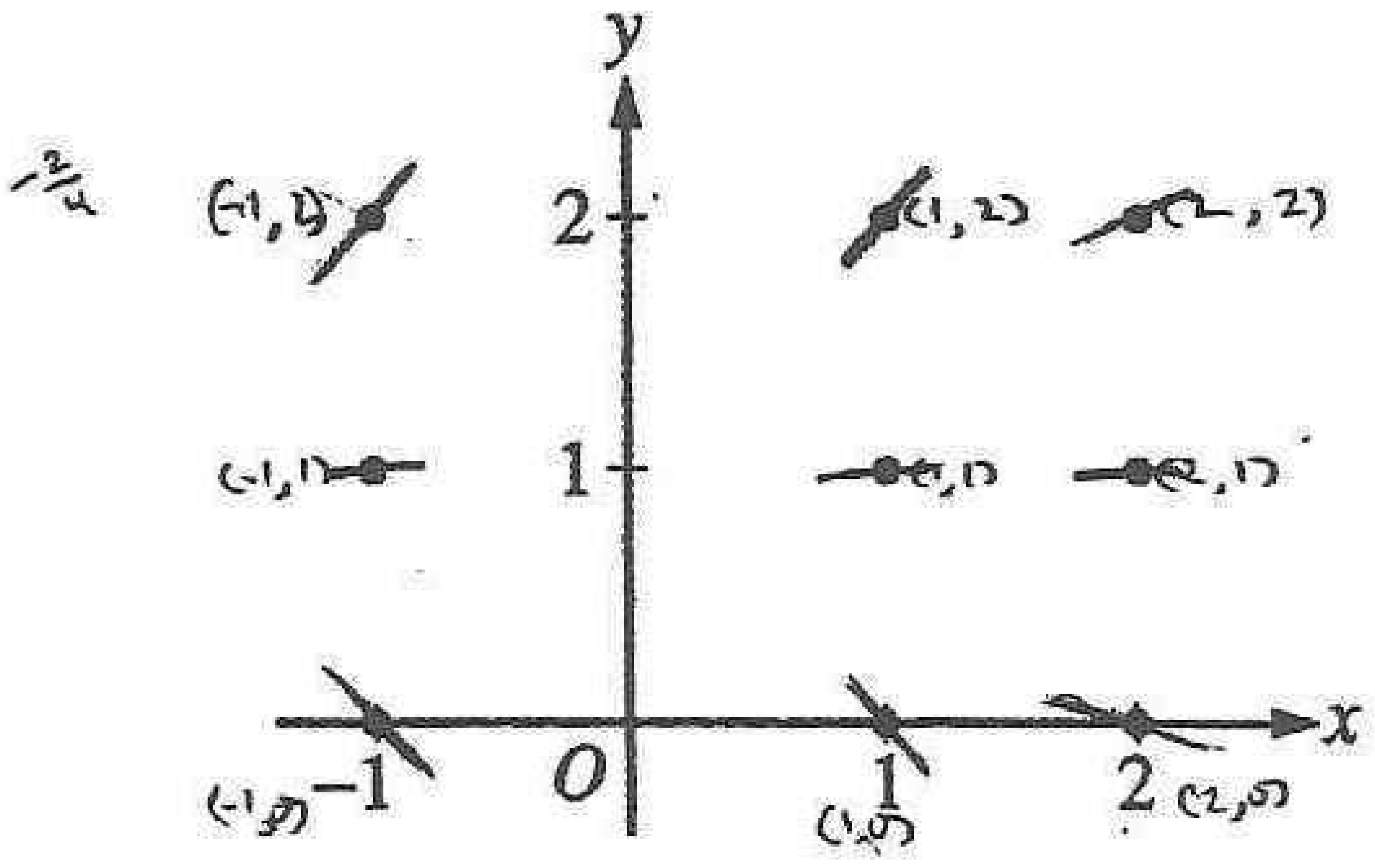
(c) $\lim_{x\to\infty} 1 - e^{\left(\frac{1}{2}-\frac{1}{x}\right)} = 1 - \sqrt{e}$

1 : limit

Sample Student Responses for Calculus AB Question 5

Student Response 1 (Score: 9)

Work for problem 5(a)



(-1,2) $\frac{(2)-1}{(-1)^2} = 1$ (1,2) $\frac{(2)-1}{(1)^2} = 1$ (2,2) $\frac{2-1}{2^2} = \frac{1}{4}$

(-1,1) $\frac{(1)-1}{(-1)^2} = 0$ (1,1) $\frac{(1)-1}{1^2} = 0$ (2,1) $\frac{1-1}{2^2} = 0$

(-1,0) $\frac{(0)-1}{(-1)^2} = -1$ (1,0) $\frac{(0)-1}{1^2} = -1$ (2,0) $\frac{0-1}{2^2} = -\frac{1}{4}$

Student Response 1 *(continued)*

Work for problem 5(b)

$\frac{dy}{dx} = \frac{y-1}{x^2}$ $\quad \frac{1}{y-1}\,dy = \frac{1}{x^2}\,dx \Rightarrow \int \frac{1}{y-1}\,dy = \int \frac{1}{x^2}\,dx$

$u = y-1$

$du = 1\,dy$

$$\int \frac{1}{u}\,du = \int x^{-2}\,dx$$

$$\ln|u| = -x^{-1} + C_0$$

$$\ln|y+1| = -\frac{1}{x} + C_0$$

$$\ln|y-1| = -\frac{1}{x} + C_0$$

$$e^{\ln|y-1|} = e^{\left(-\frac{1}{x}+C_0\right)}$$

$$|y-1| = e^{-1/x}\cdot \underbrace{e^{C_0}}_{\text{constant} = C}$$

let C absorb the absolute value bars

$y-1 = e^{-1/x}\cdot C \Rightarrow y = Ce^{-1/x} + 1 \Rightarrow 0 = C\cdot e^{-1/2} + 1 \Rightarrow C\cdot e^{-1/2} = -1$

$C = -\frac{1}{e^{-1/2}} = -e^{1/2}$

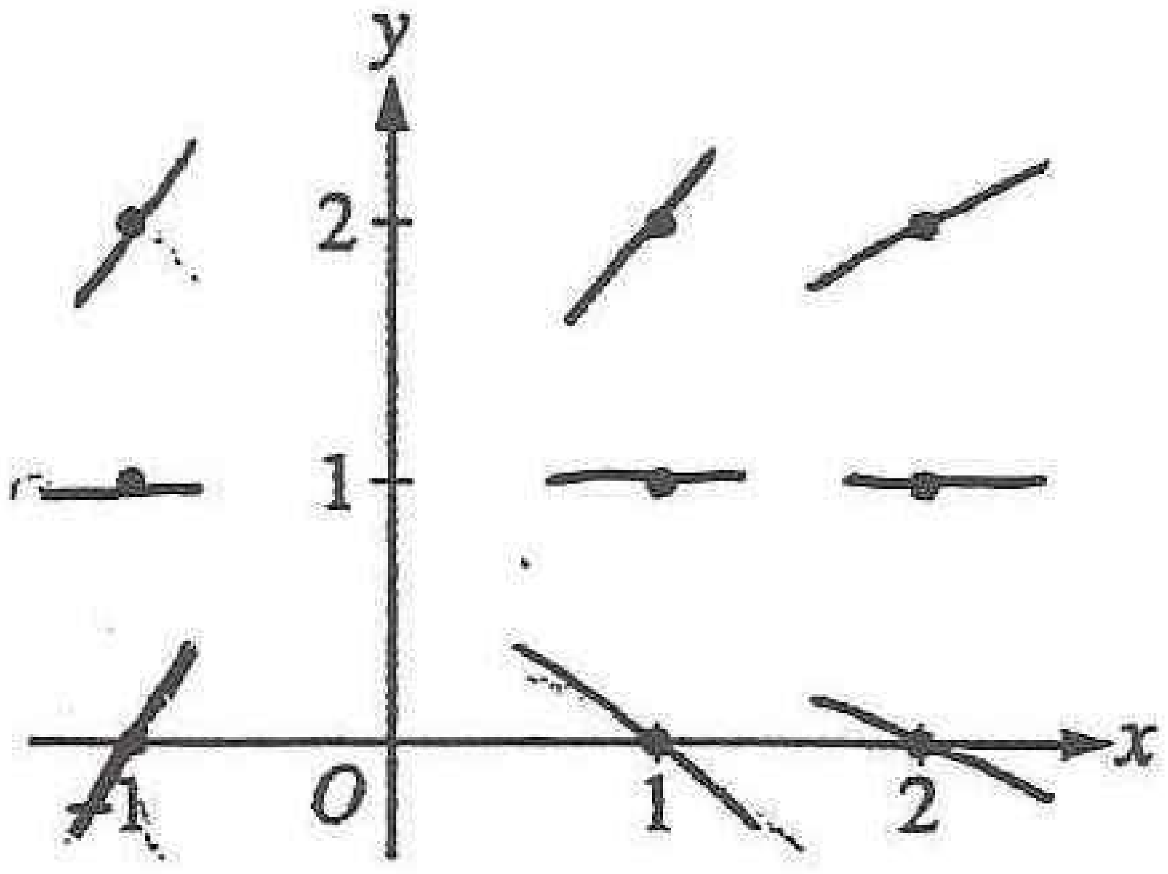
$\Rightarrow y = -e^{1/2}\cdot e^{-1/x} + 1 = -e^{\left(\frac{1}{2}-\frac{1}{x}\right)} + 1$

Work for problem 5(c)

$$\lim_{x\to\infty} f(x) = -e^{1/2}\cdot \underbrace{e^{0}}_{1} + 1 = \boxed{-e^{1/2}+1}$$

Student Response 2 (Score: 6)

Work for problem 5(a)



$$\frac{dy}{dx} = \frac{y-1}{x^2}$$

$(-1,2) \quad \frac{2-1}{1} = 1$

$(-1,1) = \frac{1-1}{1} = 0$

$(-1,0) = \frac{-1-1}{1} = \frac{2}{1} = 2$

$(1,2) = \frac{2-1}{1} = 1$

$(2,2) = \frac{2-1}{4} = \frac{1}{4}$

$(1,1) = 0$

$(1,2) = 0$

$(1,0) = \frac{-1}{1} = -1$

$(2,0) = -\frac{1}{4}$

Work for problem 5(b)

$$\frac{dy}{y-1} = \frac{dx}{x^2}$$

$$\int \frac{dy}{y-1} = \int \frac{dx}{x^2}$$

$u = y-1$

$\frac{du}{dy} = 1$

$du = dy$

$$\int \frac{du}{u} = \int \frac{dx}{x^2} \longrightarrow$$

$$\ln|u| = \frac{1}{-x} + C$$

$$\ln|y-1| = \frac{1}{-x} + C$$

$$e^{\ln|y-1|} = e^{-\frac{1}{x} + C}$$

$$|y-1| = e^{-\frac{1}{x}} \cdot e^{C} \qquad e^{C} = K$$

$$|0-1| = e^{-\frac{1}{2}} \cdot K$$

$$1 = \frac{1}{\sqrt{e}} \cdot K$$

$$\frac{1}{\frac{1}{\sqrt{e}}} = K$$

$$\sqrt{e} = K$$

$$|y-1| = e^{-\frac{1}{x}} \cdot \sqrt{e}$$

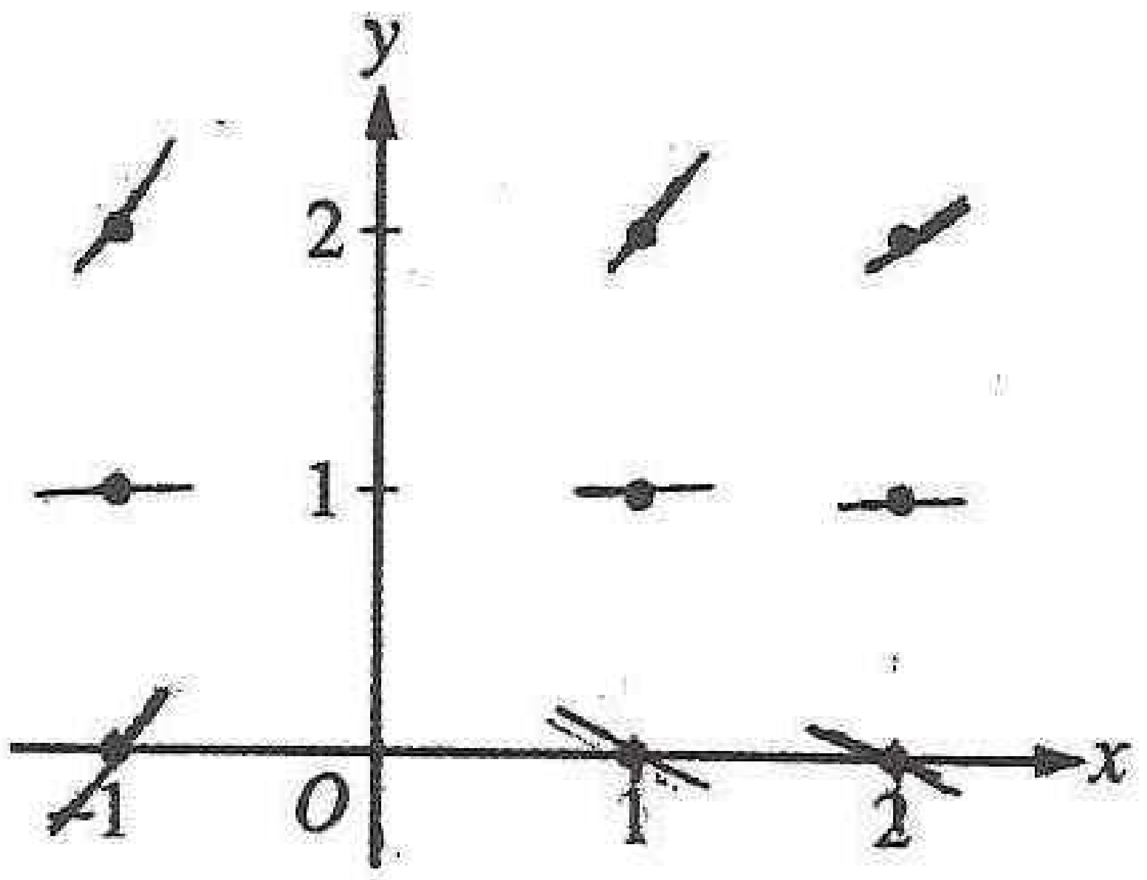
$$\boxed{y = e^{-\frac{1}{x}} \cdot \sqrt{e} + 1}$$

Work for problem 5(c)

$$\lim_{x \to \infty} e^{-\frac{1}{x}} \cdot \sqrt{e} + 1 = \qquad \underset{\to 0}{\cancel{\frac{\sqrt{e}}{e^{\frac{1}{x}}}}} + 1$$

$$\boxed{1}$$

Work for problem 5(a)



$$\frac{dy}{dx} = \frac{y-1}{x^2}$$

Student Response 3 *(continued)*

Work for problem 5(b)

$f(2)=0$

$$\frac{dy}{dx} = \frac{y^{-1}}{x^2}$$

$$dy = \left(\frac{y-1}{x2}\right)dx$$

$$\int \frac{dy}{y^{-1}} \int \frac{1}{x^2}dx$$

$$\frac{-1}{(y-1)^2} = \frac{-1}{3x^3} + C$$

$$\frac{-1}{y^2-2y+1} = \frac{-1}{3x^3} + C$$

$$\frac{-1}{0^2-2(0)+1} = \frac{-1}{3(2)^3} + C$$

$$-1 = \frac{1}{24} + C$$

$$\frac{-24}{24} - \frac{1}{24} = c$$

$$\frac{-25}{24} = C$$

$$\frac{-1}{y^2-2y+1} = \frac{-1}{2x^3}$$

$$-3x^3 = y^2-2y+1$$

$$-3x^3-1 = y^2-2y$$

$$\frac{-3x^3-1}{2} = y^2$$

$$y = \sqrt{\frac{-3x^3-1}{2}}$$

$$y = \sqrt{\frac{-3x^3 - \frac{25}{24}}{2}}$$

Work for problem 5(c)

$$\lim_{x \to \infty} f(x) = -\infty$$

Calculus AB Question 6

Overview

This problem presented students with a function f defined by $f(x) = \frac{\ln x}{x}$ for $x > 0$, together with a formula for $f'(x)$. Part (a) asked for an equation of the line tangent to the graph of f at $x = e^2$. In part (b) students needed to solve $f'(x) = 0$ and determine the character of this critical point from the supplied $f'(x)$. In part (c) students had to demonstrate skill with the quotient rule to obtain a formula for $f''(x)$ and solve $f''(x) = 0$ to find the x-coordinate of what was promised to be the only point of inflection for the graph of f. Part (d) tested students' knowledge of properties of $\ln x$ to determine the limit of $f(x)$ as $x \to 0^+$.

The mean score for this question was 3.06. The question included several arithmetic and algebraic calculations that can prove difficult for students. In part (a) students needed to find the values of both $f\left(e^2\right)$ and $f'\left(e^2\right)$ in order to earn the first point. The second point was earned for a correct tangent line equation. Students should be careful not to confuse y with $f(x)$ here. Otherwise they are making the claim that the function f is linear. In part (b) a local argument was sufficient. Students earned the first 2 points with correct answers, even without a correct justification. In part (c) if a student made an error in applying the quotient rule, the student did not earn either of the 2 derivative points. In this same part, the problem stated that there is a single point of inflection for $x > 0$. Since f is twice differentiable for $x > 0$, knowing that $f''(x) = 0$ only at $x = e^{3/2}$ is sufficient to conclude that $x = e^{3/2}$ is the x-coordinate of the point of inflection. In part (d), while it is true that the limit does not exist, saying that the limit is $-\infty$ is also true and is more specific. Either answer earned the point.

Commentary on Student Responses

Student Response 1 (Score: 9)

The student earned all 9 points.

Student Response 2 (Score: 6)

The student earned 6 points: 1 point in part (a), 3 points in part (b), 2 points in part (c), and no points in part (d). In part (a) the student identifies the value of the function and the derivative at $x = e^2$, which earned the first point. The student does not simplify these values correctly, and the second point was not earned. In part (b) the student correctly identifies $x = e$, classifies $x = e$ as a maximum, and justifies that answer. In part (c) the student earned 2 points for correctly using the quotient rule to find the second derivative. The student does not solve the equation to find the x-coordinate of the point of inflection and did not earn the third point. In part (d) the student's answer is not correct.

Student Response 3 (Score: 4)

The student earned 4 points: 1 point in part (a), 1 point in part (b), 2 points in part (c), and no points in part (d). In part (a) the student identifies the value of the function and the derivative at $x = e^2$, which earned the first point. The student makes an arithmetic error in simplifying these values, and the second point was not earned. In part (b) the student correctly identifies $x = e$, but the student classifies it as neither a minimum nor a maximum. Only the first point was earned. In part (c) the student earned 2 points for correctly using the quotient rule to find the second derivative. The student makes two arithmetic errors in solving the equation and did not earn the third point. In part (d) the student's answer is not correct.

Scoring Guidelines for Calculus AB Question 6

Let f be the function given by $f(x) = \dfrac{\ln x}{x}$ for all $x > 0$. The derivative of f is given by $f'(x) = \dfrac{1 - \ln x}{x^2}$.

(a) Write an equation for the line tangent to the graph of f at $x = e^2$.

(b) Find the x-coordinate of the critical point of f. Determine whether this point is a relative minimum, a relative maximum, or neither for the function f. Justify your answer.

(c) The graph of the function f has exactly one point of inflection. Find the x-coordinate of this point.

(d) Find $\lim\limits_{x \to 0^+} f(x)$.

(a) $f\left(e^2\right) = \dfrac{\ln e^2}{e^2} = \dfrac{2}{e^2}$, $f'\left(e^2\right) = \dfrac{1 - \ln e^2}{\left(e^2\right)^2} = -\dfrac{1}{e^4}$ An equation for the tangent line is $y = \dfrac{2}{e^2} - \dfrac{1}{e^4}\left(x - e^2\right)$.	$2 : \begin{cases} 1 : f\left(e^2\right) \text{ and } f'\left(e^2\right) \\ 1 : \text{answer} \end{cases}$
(b) $f'(x) = 0$ when $x = e$. The function f has a relative maximum at $x = e$ because $f'(x)$ changes from positive to negative at $x = e$.	$3 : \begin{cases} 1 : x = e \\ 1 : \text{relative maximum} \\ 1 : \text{justification} \end{cases}$
(c) $f''(x) = \dfrac{-\frac{1}{x}x^2 - (1 - \ln x)2x}{x^4} = \dfrac{-3 + 2\ln x}{x^3}$ for all $x > 0$ $f''(x) = 0$ when $-3 + 2\ln x = 0$ $x = e^{3/2}$ The graph of f has a point of inflection at $x = e^{3/2}$ because $f''(x)$ changes sign at $x = e^{3/2}$.	$3 : \begin{cases} 2 : f''(x) \\ 1 : \text{answer} \end{cases}$
(d) $\lim\limits_{x \to 0^+} \dfrac{\ln x}{x} = -\infty$ or Does Not Exist	1 : answer

Sample Student Responses for Calculus AB Question 6

Student Response 1 (Score: 9)

Work for problem 6(a)

$x = e^2 \quad \left(e^2, \frac{2}{e^2}\right)$

$$f(e^2) = \frac{\ln(e^2)}{e^2} = \frac{2}{e^2}$$

$$\frac{x\left(\frac{1}{x}\right) - \ln x}{x^2}$$

$$f'(e^2) = \frac{1 - \ln(e^2)}{(e^2)^2} = \frac{1-2}{e^4} = \frac{-1}{e^4} = m$$

$$\boxed{y - \frac{2}{e^2} = \frac{-1}{e^4}\left(x - e^2\right)}$$

Work for problem 6(b)

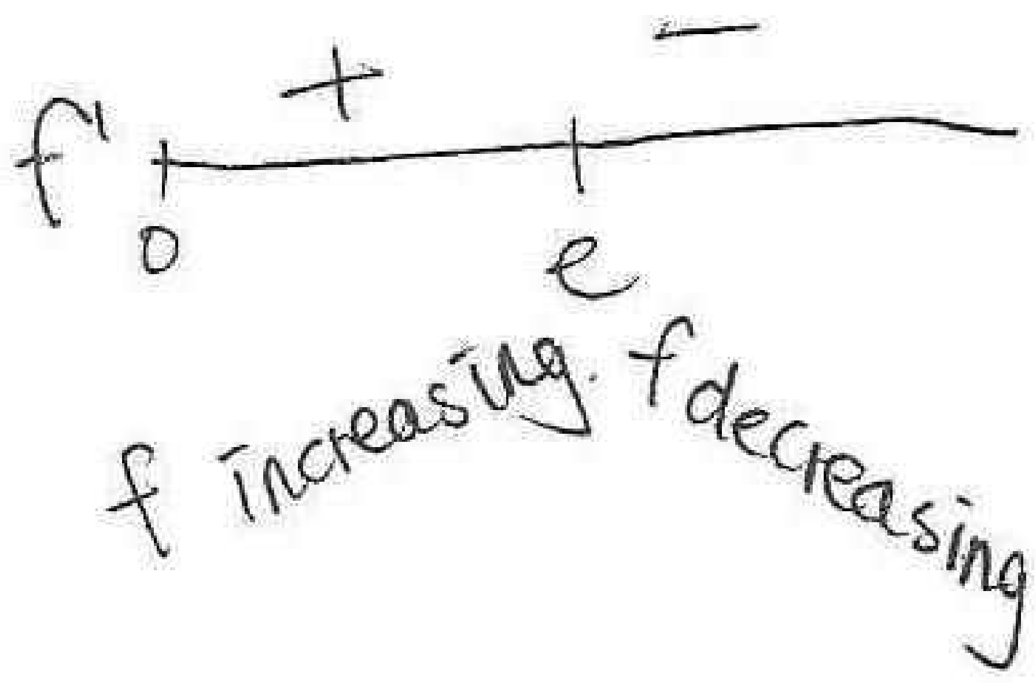
$$f'(x) = 0 = \frac{1 - \ln x}{x^2}$$

$0 = 1 - \ln x$

$\ln x = 1$

$\ln e = 1$

$\boxed{x = e}$



f has relative maximum at $x = e$ since f' changes signs from positive to negative

Student Response 1 *(continued)*

Work for problem 6(c)

$$f'(x)=\frac{1-\ln x}{x^2}$$

$$f''(x)=\frac{(x^2)\left(\frac{-1}{\cancel{x}}\right)-(1-\ln x)(2x)}{x^4}$$

$$=\frac{-x-(2x-2x\ln x)}{x^4}$$

$$=\frac{-x-2x+2x\ln x}{x^4}$$

$$=\frac{-3x+2x\ln x}{x^4}$$

$$0=\frac{-3x+2x\ln x}{x^4}=f''(x)$$

$$0=-3x+2x\ln x$$

$$3\cancel{x}=2\cancel{x}\ln x$$

$$3=2\ln x$$

$$\frac{3}{2}=\ln x$$

$$\boxed{x=e^{3/2}}$$

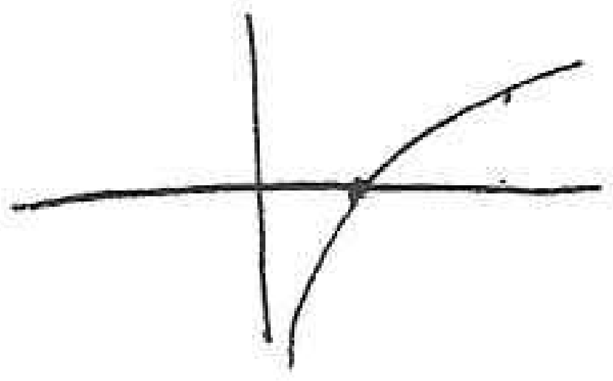
Work for problem 6(d)

0.001

$$\lim_{x\to 0^+}\frac{\ln x}{x}$$

$$\frac{-\infty}{0.001}$$

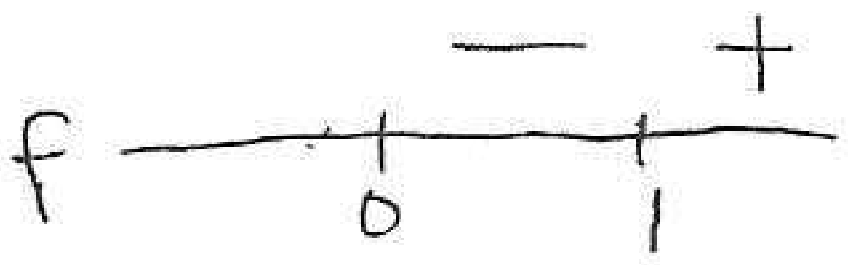
$$=\boxed{-\infty}$$



e^{-1} $\quad \frac{-1}{e^{-1}}$ $\quad -999e^{999}$

e^{-999} $\quad \frac{-999}{e^{-999}}$

x	y
e^{-1}	$\frac{-1}{e^{-1}}=-e$
e^{-999}	$\frac{-999}{e^{-999}}=-999e^{999}$
e^{-n}	$\frac{-n}{e^{-n}}=-ne^{n}$



$\rightarrow$ limit of $f(x)$ approaches $-\infty$ as $x\to 0^+$

Student Response 2 (Score: 6)

Work for problem 6(a)

$$f'(e^2) = \frac{1-\ln e^2}{e^4}$$

$$= \frac{1-2\ln e}{e^4}$$

$$= \frac{1-2e}{e^4}$$

$$f(e^2) = \frac{\ln e^2}{e^2}$$

$$= \frac{2\ln e}{e^2}$$

$$= \frac{2e}{e^2} = \frac{2}{e}$$

$$y - y_1 = m(x - x_1)$$

$$y - \frac{2}{e} = \frac{1-2e}{e^4}(x - e^2)$$

$$y - \frac{2}{e} = \frac{x - e^2 - 2ex + 2e^3}{e^4}$$

$$-2e^3$$

$$y = \frac{x + 2e^3 - e^2 - 2ex}{e^4} + \frac{2}{e}$$

$$= \frac{x + 2ex + 4e^3 - e^2}{e^4}$$

Work for problem 6(b)

$$f'(x) = \frac{1-\ln x}{x^2} = 0$$

$$1 - \ln x = 0$$

$$\ln x = 1$$

$$x = e$$

Criticle number at $x=e$

relative maximum because the graph of $f'(x)$ is positive before $x=e$, then 0 at $x=e$ and negative after $x=e$

Student Response 2 *(continued)*

Work for problem 6(c)

$$f''(x) = \frac{x^2\left(-\frac{1}{x}\right) - (1-\ln x)(2x)}{x^4}$$

$$= \frac{-x - 2x + 2x\ln x}{x^4} = 0$$

$$x(-1-2+2\ln x) = 0$$

$$x = 0 \quad \Big| \quad -3 + 2\ln x = 0$$

$$\ln x = 3/2$$

POI at $x = 0$

Work for problem 6(d)

$$\lim_{x \to 0^+} \left(\frac{\ln x}{x}\right)$$

$$\rightarrow \infty$$

Student Response 3 (Score: 4)

Work for problem 6(a)

$f(x) = \frac{\ln x}{x}$

$f'(x) = \frac{1 - \ln x}{x^2}$

$x = e^2$

$f(e^2) = \frac{\overset{e}{\cancel{\ln e^2}}}{e^2} \Rightarrow \frac{\cancel{A}}{e^{\cancel{x}}} = \frac{1}{e}$

$(e^2, 1/e)$ point

$\frac{1 - \overset{e}{\cancel{\ln e^2}}}{(e^2)^2}$

$\frac{1-e}{e^4}$ = slope

$$\boxed{y - 1/e = \frac{1-e}{e^4}(x - e^2)}$$

Work for problem 6(b)

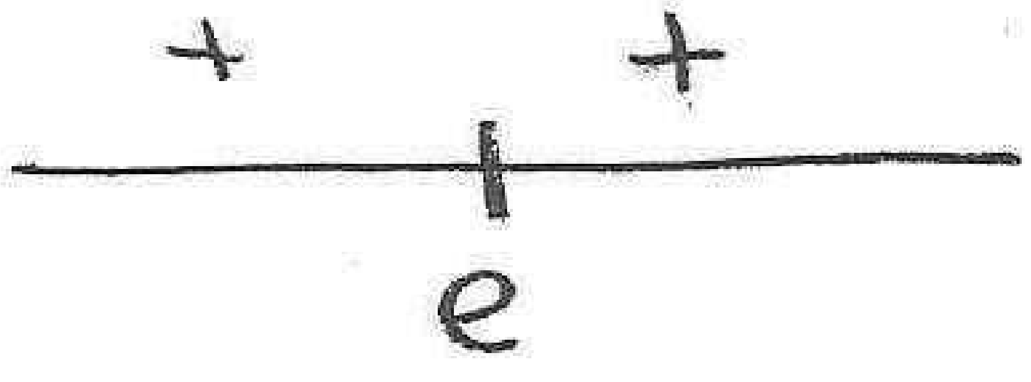
$0 = \frac{1 - \ln x}{x^2}$

$0 = 1 - \ln x$

$+1 = +\ln x$

$e^1 = e^{\cancel{\ln} x}$

$\boxed{e = x}$



the point
∧ is neither b.c
the function is increasing
on both sides.

Student Response 3 *(continued)*

Work for problem 6(c)

$$f'(x) = \frac{1 - \ln x}{x^2}$$

$$f''(x) = \frac{x^{\cancel{2}}\left(-\frac{1}{\cancel{x}}\right) - (1 - \ln x)(2x)}{(x^2)^2}$$

$$0 = \frac{-x - 2x - 2x \ln x}{x^4}$$

$$\boxed{e^{3/2} = x}$$

$$0 = -3x - 2x \ln x$$

$$\frac{3x}{2x} = \frac{2x \ln x}{2x}$$

$$e\frac{3}{2} = \cancel{e\ln}\, x$$

Work for problem 6(d)

$$\lim_{x \to 0^+} \frac{\ln x}{x} \qquad \frac{e}{1}$$

$$\boxed{e}$$

Calculus BC Question 3

Overview

This problem presented students with a table of values for a function h and its derivatives up to the fourth order at $x = 1$, $x = 2$, and $x = 3$. The question stated that h has derivatives of all orders, and that the first four derivatives are increasing on $1 \le x \le 3$. Part (a) asked for the first-degree Taylor polynomial about $x = 2$ and the approximation for $h(1.9)$ given by this polynomial. Students needed to use the given information to determine that the graph of h is concave up between $x = 1.9$ and $x = 2$ to conclude that this approximation is less than the value of $h(1.9)$. Part (b) asked for the third-degree Taylor polynomial about $x = 2$ and the approximation for $h(1.9)$ given by this polynomial. In part (c) students were expected to observe that the given conditions imply that $\left|h^{(4)}(x)\right|$ is bounded above by $h^{(4)}(2)$ on $1.9 \le x \le 2$ and apply this to the Lagrange error bound to show that the estimate in part (b) has error less than 3×10^{-4}.

The mean score for this question was 4.42. The focus of the problem was estimation using Taylor polynomials. The ability to determine whether an estimate is too large or too small and being able to constrain the error in an estimate are important skills. In parts (a) and (b), the Taylor polynomials could be constructed by taking values for f and its derivatives at $x = 2$ from the table. When reading values from a table, students should be careful to select the values they intend. In part (a) students need to include their reasoning as to why their estimate was an underestimate in order to earn the last point. In part (c), in order to earn the last point, students should clearly indicate that $\left|h^{(4)}(x)\right| \le \frac{584}{9}$ since $h^{(4)}$ is increasing throughout the interval. In general students need practice working with the Lagrange error bound and with other error bounds related to series convergence.

Commentary on Student Responses

Student Response 1 (Score: 9)

The student earned all 9 points.

Student Response 2 (Score: 6)

The student earned 6 points: 3 points in part (a), 2 points in part (b), and 1 point in part (c). In part (a) the student gives a correct linear polynomial and a correct evaluation of $P_1(1.9)$ and earned the first 3 points. The student states that the approximation is less than $h(1.9)$ but only mentions that $h''(2) > 0$. An argument at a point is not sufficient to earn the last point. In part (b) the student's polynomial has an error in the coefficient of the cubic term and earned 1 point. The student correctly evaluates $P_3(1.9)$ from the student's polynomial and earned the last point. In part (c) the student has the proper form for the Lagrange error term and earned the first point. The student does not properly bound the fourth derivative and did not earn the last point.

Student Response 3 (Score: 4)

The student earned 4 points: 2 points in part (a), 2 points in part (b), and no points in part (c). In part (a) the student gives a correct linear polynomial and earned 2 points. The evaluation of $P_1(1.9)$ is not correct and did not earn the third point. The student states that $P_1(1.9) < h(1.9)$ but does not provide a valid argument. The last point was not earned. In part (b) the student has a correct polynomial and earned both points. The student does not correctly evaluate $P_3(1.9)$ and did not earn the last point.

Scoring Guidelines for Calculus BC Question 3

x	$h(x)$	$h'(x)$	$h''(x)$	$h'''(x)$	$h^{(4)}(x)$
1	11	30	42	99	18
2	80	128	$\frac{488}{3}$	$\frac{448}{3}$	$\frac{584}{9}$
3	317	$\frac{753}{2}$	$\frac{1383}{4}$	$\frac{3483}{16}$	$\frac{1125}{16}$

Let h be a function having derivatives of all orders for $x > 0$. Selected values of h and its first four derivatives are indicated in the table above. The function h and these four derivatives are increasing on the interval $1 \le x \le 3$.

(a) Write the first-degree Taylor polynomial for h about $x = 2$ and use it to approximate $h(1.9)$. Is this approximation greater than or less than $h(1.9)$? Explain your reasoning.

(b) Write the third-degree Taylor polynomial for h about $x = 2$ and use it to approximate $h(1.9)$.

(c) Use the Lagrange error bound to show that the third-degree Taylor polynomial for h about $x = 2$ approximates $h(1.9)$ with error less than 3×10^{-4}.

(a) $P_1(x) = 80 + 128(x-2)$, so $h(1.9) \approx P_1(1.9) = 67.2$

$P_1(1.9) < h(1.9)$ since h' is increasing on the interval $1 \le x \le 3$.

$$4 : \begin{cases} 2 : P_1(x) \\ 1 : P_1(1.9) \\ 1 : P_1(1.9) < h(1.9) \text{ with reason} \end{cases}$$

(b) $P_3(x) = 80 + 128(x-2) + \frac{488}{6}(x-2)^2 + \frac{448}{18}(x-2)^3$

$h(1.9) \approx P_3(1.9) = 67.988$

$$3 : \begin{cases} 2 : P_3(x) \\ 1 : P_3(1.9) \end{cases}$$

(c) The fourth derivative of h is increasing on the interval $1 \le x \le 3$, so $\max\limits_{1.9 \le x \le 2} \left| h^{(4)}(x) \right| = \frac{584}{9}$.

$$\text{Therefore, } |h(1.9) - P_3(1.9)| \le \frac{584}{9} \frac{|1.9-2|^4}{4!}$$
$$= 2.7037 \times 10^{-4}$$
$$< 3 \times 10^{-4}$$

$$2 : \begin{cases} 1 : \text{form of Lagrange error estimate} \\ 1 : \text{reasoning} \end{cases}$$

Sample Student Responses for Calculus BC Question 3

Student Response 1 (Score: 9)

x	$h(x)$	$h'(x)$	$h''(x)$	$h'''(x)$	$h^{(4)}(x)$
1	11	30	42	99	18
2	80	128	$\frac{488}{3}$	$\frac{448}{3}$	$\frac{584}{9}$
3	317	$\frac{753}{2}$	$\frac{1383}{4}$	$\frac{3483}{16}$	$\frac{1125}{16}$

Work for problem 3(a)

$$\boxed{P_1(x) = 80 + 128(x-2)}$$

$$h(1.9) \approx P_1(1.9) = 80 + 128(-.1) = \boxed{67.2}$$

This approximation is less than h(1.9) because since h'(x) is increasing, h''(x) > 0 and h(x) is concave up.

Work for problem 3(b)

$$P_3(x) = 80 + 128(x-2) + \frac{244}{3}(x-2)^2 + \frac{224}{9}(x-2)^3$$

$$h(1.9) \approx P_3(1.9) = 80 + 128(-.1) + \frac{244}{3}(-.1)^2 + \frac{224}{9}(-.1)^3 \approx \boxed{67.988}$$

Work for problem 3(c)

R

Rn(X) denotes the remainder, also known as the Lagrange error.

$$R_n(x) = \frac{f^{(n+1)}(c)}{(n+1)!}(x-a)^{n+1}$$

$\boxed{a=2, \; n=3, \; x=1.9}$ → $R_3(1.9) = \frac{f^{(4)}(c)}{24}(+.1)^4$ for some c between 1.9 and 2

$$= .00000417\, f^{(4)}(c)$$

* We know that $f^{(4)}(c)$ is increasing, so for $1.9 < c < 2$, $18 < f^{(4)}(c) < \frac{584}{9}$.

$$.0000751 < .00000417 f^{(4)}(c) < .000271$$

* $R_3(1.9) = .00000417 f^{(4)}(c) < .000271$ and $.000271 < .0003$, so by transitivity $R_3(1.9) < .0003$.

Student Response 2 (Score: 6)

x	$h(x)$	$h'(x)$	$h''(x)$	$h'''(x)$	$h^{(4)}(x)$
1	11	30	42	99	18
2	80	128	$\frac{488}{3}$	$\frac{448}{3}$	$\frac{584}{9}$
3	317	$\frac{753}{2}$	$\frac{1383}{4}$	$\frac{3483}{16}$	$\frac{1125}{16}$

Work for problem 3(a)

$P_1 = 80 + 128(x-2)$

$h(1.9) \approx 80 + 128(1.9-2) = 67.2$

This approxamation is less than $h(1.9)$ because [illegible] $h''(2) > 0$.

Student Response 2 *(continued)*

Work for problem 3(b)

$$P_3 = 80 + 128(x-2) + \frac{244}{3}(x-2)^2 + \frac{244}{9}(x-2)^3$$

$$h(1.9) \approx 67.986$$

Work for problem 3(c)

$$|S - S_n| \le \frac{M}{(n+1)!}|x-2|^{n+1} = \frac{\frac{584}{9}}{4!}|1.9-2|^4 < 3 \times 10^{-4}$$

Student Response 3 (Score: 4)

x	$h(x)$	$h'(x)$	$h''(x)$	$h'''(x)$	$h^{(4)}(x)$
1	11	30	42	99	18
2	80	128	$\frac{488}{3}$	$\frac{448}{3}$	$\frac{584}{9}$
3	317	$\frac{753}{2}$	$\frac{1383}{4}$	$\frac{3483}{16}$	$\frac{1125}{16}$

$$f(x) = f(a) + f'(a)(x-a) + \frac{f''(a)(x-a)^2}{2!} + \dots$$

Work for problem 3(a)

$$P_3 = h(2) + h'(2)(x-2)$$

$$= 80 + 128(x-2)$$

$$h(1.9) \approx 80 + 128(x-2)$$

$$= \overset{7}{\underset{-12}{80}} - 12.8$$

$$= 68.2$$

this less than h(1.9) because Taylor series is endless we are using only few of the series to approximate h(1.9).

h(a) + f

80+

1.9

$x-1.9)$

5 [illegible] 6 38

Student Response 3 *(continued)*

Work for problem 3(b)

$$P_3 = h(2) + h'(2)(x-2) + \frac{h''(2)(x-2)^2}{2!} + \frac{h'''(2)(x-2)^3}{3!}$$

$$= 80 + 128(x-2) + \frac{488(x-2)^2}{6} + \frac{448(x-2)^3}{18}$$

$$h(1.9) \approx 80 + 128(x-2) + \frac{488(x-2)^2}{6} + \frac{448(x-2)^3}{18}$$

$$= \boxed{80 - 12.8} + 0.813 - 0.253$$

$$=$$

Work for problem 3(c)

— — — Lagrange

~~M | < t~~

Calculus BC Question 5

Overview

In this problem, students were told that a function *f* has derivative $f'(x) = (x - 3)e^x$ and that $f(1) = 7$. In part (a) students needed to determine with justification the character of the critical point for *f* at $x = 3$. Part (b) asked for the intervals on which the graph of *f* is both decreasing and concave up. For this, students had to apply the product rule to obtain $f''(x)$. In part (c) students needed to solve the initial value problem to find $f(3)$, employing integration by parts along the way.

The mean score for this question was 5.82. Both parts (a) and (b) required reasoning in order for students to earn the last points. In both cases sign charts by themselves were insufficient reasoning. Charts attempting to indicate the sign of a function or derivative need to be explained with mathematical notation (e.g., $f'(x) < 0$ for $0 < x < 3$) or in words. In part (c) students either needed to show their work of integrating by parts, or they had to present an antiderivative of $f'(x)$ and verify through differentiation that it is indeed an antiderivative. In general, students need facility with many methods of antidifferentiation, including integration by parts.

Commentary on Student Responses

Student Response 1 (Score: 9)

The student earned all 9 points. In part (a) the student correctly identifies $x = 3$ as a relative minimum and justifies the answer using the First Derivative Test. In part (b) the student correctly finds the second derivative using the product rule and the correct interval. The student explains the reasoning for the answer. In part (c) the student correctly integrates using a tabular method for integration by parts. The student uses the initial condition and has a correct answer.

Student Response 2 (Score: 6)

The student earned 6 points: 1 point in part (a), 2 points in part (b), and 3 points in part (c). In part (a) the student correctly identifies $x = 3$ as a relative minimum but does not connect the justification for the answer to the derivative. In part (b) the student correctly finds the second derivative but does not connect the explanation to the first or second derivatives. In part (c) the student earned the integration by parts points and uses the initial condition correctly. The student makes a simplification error and did not earn the answer point.

Student Response 3 (Score: 3)

The student earned 3 points: 1 point in part (a), 2 points in part (b), and no points in part (c). In part (a) the student correctly identifies $x = 3$ as a relative minimum but does not connect the justification for the answer to the derivative. In part (b) the student finds the correct second derivative but does not give the correct interval answer. In part (c) the student does not have an integral expression and was not eligible to earn the initial condition or answer points.

Scoring Guidelines for Calculus BC Question 5

The derivative of a function f is given by $f'(x) = (x-3)e^x$ for $x > 0$, and $f(1) = 7$.

(a) The function f has a critical point at $x = 3$. At this point, does f have a relative minimum, a relative maximum, or neither? Justify your answer.

(b) On what intervals, if any, is the graph of f both decreasing and concave up? Explain your reasoning.

(c) Find the value of $f(3)$.

(a) $f'(x) < 0$ for $0 < x < 3$ and $f'(x) > 0$ for $x > 3$

Therefore, f has a relative minimum at $x = 3$.

$$2 : \begin{cases} 1\text{: minimum at } x = 3 \\ 1\text{: justification} \end{cases}$$

(b) $f''(x) = e^x + (x-3)e^x = (x-2)e^x$

$f''(x) > 0$ for $x > 2$

$f'(x) < 0$ for $0 < x < 3$

Therefore, the graph of f is both decreasing and concave up on the interval $2 < x < 3$.

$$3 : \begin{cases} 2 : f''(x) \\ 1 : \text{answer with reason} \end{cases}$$

(c) $f(3) = f(1) + \int_1^3 f'(x)\,dx = 7 + \int_1^3 (x-3)e^x\,dx$

$$u = x - 3 \quad dv = e^x\,dx$$

$$du = dx \quad v = e^x$$

$$f(3) = 7 + (x-3)e^x\Big|_1^3 - \int_1^3 e^x\,dx$$

$$= 7 + \left((x-3)e^x - e^x\right)\Big|_1^3$$

$$= 7 + 3e - e^3$$

$$4 : \begin{cases} 1 : \text{uses initial condition} \\ 2 : \text{integration by parts} \\ 1 : \text{answer} \end{cases}$$

Sample Student Responses for Calculus BC Question 5

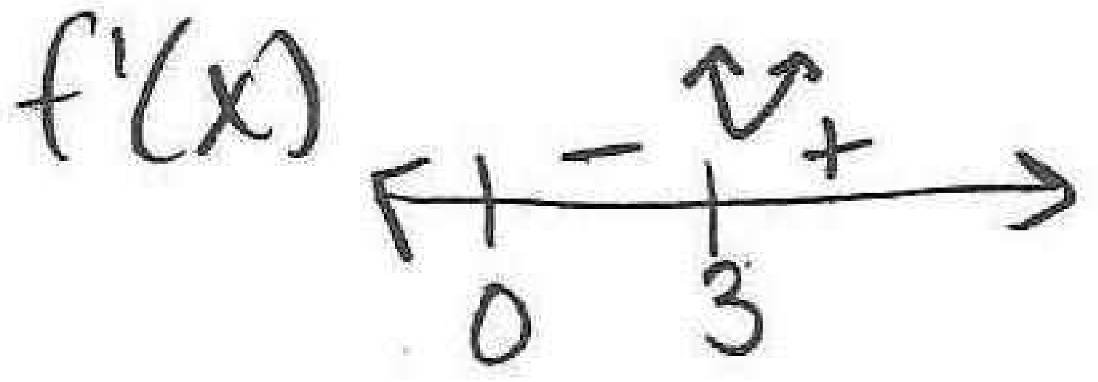
Student Response 1 (Score: 9)

Work for problem 5(a)

$f'(x) = (x-3)e^x$ $\quad x > 0$

$f(1) = 7$

$f'(3) = 0$ and the function has a critical point



Because $f'(x) = 0$ and $f'(x)$ changes sign from negative to positive at $x = 3$, the function $f(x)$ has a relative minimum at this point.

$f'(1) = -2e$

$f'(4) = 1e^4 = e^4$

Work for problem 5(b)

$f'(x) = (x-3)e^x$

to be decreasing $f'(x)$ must be < 0

to be concave up $f''(x)$ must be > 0

$f'(x) = (x-3)e^x$

$f''(x) = (x-3)e^x + e^x(1)$

$e^x + e^x(x-3)$

$e^x(1 + (x-3))$

$f''(x) = e^x(x-2)$

$f'(x) < 0$ when $x < 3$

$f''(x) > 0$ when $x > 2$

$\therefore$ the graph of f is both decreasing and concave up when $\boxed{2 < x < 3}$ because in that interval $f'(x) < 0$ and $f''(x) > 0$.

Student Response 1 *(continued)*

Work for problem 5(c)

$f(3) = ?$

$f'(x) = (x-3)e^x \quad x>0$

$f(x) = \int (x-3)e^x dx \quad f(1) = 7$

	u	dx
+	x-3	$e^x dx$
-	1	e^x
+	0	e^x

$f(x) = (x-3)e^x - e^x + c$

$7 = -2e^1 - e^1 + c$

$7 = -2e - e + c$

$7 = -3e + c$

$7 + 3e = c$

$f(x) = (x-3)e^x - e^x + 7 + 3e$

$f(3) = (3-3)e^3 - e^3 + 7 + 3e$

$= -e^3 + 7 + 3e$

$= 7 + 3e - e^3$

$\boxed{f(3) = 7 + 3e - e^3}$

Student Response 2 (Score: 6)

Work for problem 5(a)

f has a min at $x=3$ because the slopes change from neg. to pos.

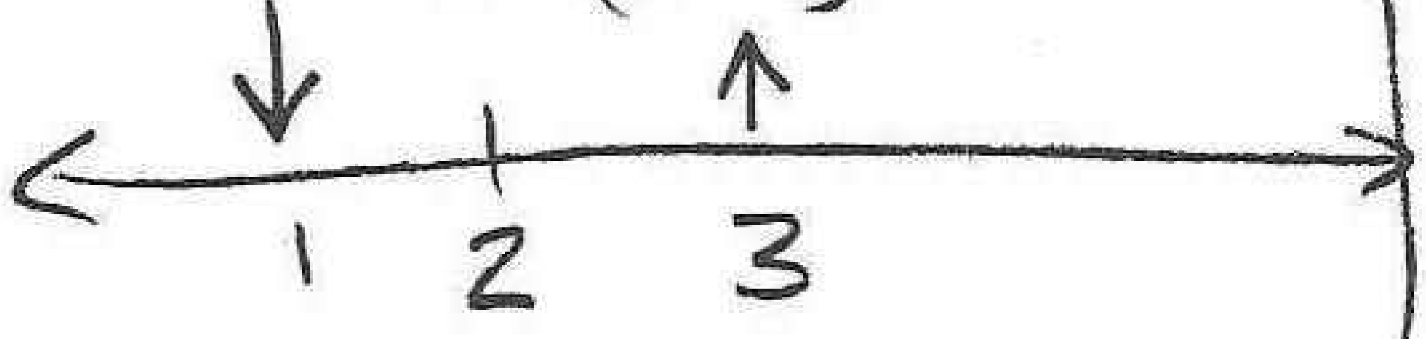
Work for problem 5(b)

$f'(x)=(x-3)e^x$

$f''(x)=(x-3)e^x+e^x$

$e^x(x-3+1)$

$e^x(x-2)$



f is conc. up for $x>2$ and dec. for $x<3$ therefore f is dec. and conc. up on the interval $\boxed{2<x<3}$

Student Response 2 *(continued)*

Work for problem 5(c)

$\int f'(x)$

$\int (x-3)e^x$

$u = x-3 \quad dv = e^x$

$du = 1 \quad v = e^x$

$(x-3)(e^x) - \int e^x$

$e^x(x-3) - e^x + C$

$e^x(x-4) + c = f(x)$

$7 = e^1(1-4) + c$

$\overline{-3} \quad \frac{-3}{-3}$

$-\frac{7}{3} = e + c$

$-e \quad -e$

$-\frac{7}{3} - e = C$

$f(x) = e^x(x-4) - \frac{7}{3} - e$

$f(3) = e^3(3-4) - \frac{7}{3} - e$

$\boxed{f(3) = -e^3 - e - \frac{7}{3}}$

Student Response 3 (Score: 3)

Work for problem 5(a)

at $x=2$ $f'(x)<0$

at $x=4$ $f'(x)>0$

therefore at $x=3$ there is a minimum because at $x=2$ $f(x)$ is decreasing to the minimum at $f(3)$ and begins increasing again, thus at $f(4)$ it is increasing.

Work for problem 5(b)

f is decreasing when $0<x<3$. f is concave up when $f''(x)>0$ $f''(x)=(x-3)e^x+e^x$. f is concave up when $0<x<3$

therefore f is decreasing and concave up

$0<x<3$

Work for problem 5(c)

$$f'(3)=0$$

$$f(x)=mx+b$$

$$f(3)=0(3)+b$$

$$\boxed{f(3)=7+\frac{2}{e}}$$

$$f'(1)=\frac{-2}{e}$$

$$f(1)=\frac{-2}{e}+b$$

$$f(1)=7=\frac{-2}{e}+b$$

$$7+\frac{2}{e}=b$$

Calculus BC Question 6

Overview

This problem presented students with a logistic differential equation and the initial value $f(0) = 8$ of a particular solution $y = f(t)$. In part (a) a slope field for the differential equation was given, and students were asked to sketch solution curves through two specified points. In particular, students should have demonstrated appropriate behavior for these curves for $t \geq 0$, especially with regard to the horizontal lines $y = 0$ and $y = 6$. For part (b) students needed to use the given initial value for the solution *f* and a two-step Euler's method to approximate $f(1)$. In part (c) students were directed to find the second-degree Taylor polynomial for *f* about $t = 0$ and use it to approximate $f(1)$. Part (d) asked for the range of the particular solution $y = f(t)$.

The mean score for this question was 3.84. This question focused on using a differential equation to determine information about solutions without actually finding solutions. In part (a) students sketch graphs of solutions with different initial conditions. It is important that these graphs do not cross the equilibrium solutions $y = 0$ and $y = 6$. In part (b) students need to clearly show the work involved in Euler's method. If students use a table to organize their work, the table must be clearly labeled. In part (c) students need to show their work in determining $\frac{d^2y}{dx^2}$. To be eligible for either of the first 2 points, a student must correctly apply both the chain rule and the product rule in computing $\frac{d^2y}{dx^2}$. In part (d) $\lim_{x\to\infty} f(x) = 6$, but 6 is not in the range of *f*. The function *h* given by $h(x) = 6$ is an equilibrium solution to the differential equation, and two solutions cannot intersect.

Commentary on Student Responses

Student Response 1 (Score: 9)

The student earned all 9 points. Note that the reason the student provides in part (d) was not required.

Student Response 2 (Score: 6)

The student earned 6 points: 2 points in part (a), 1 point in part (b), 3 points in part (c), and no points in part (d). In part (a) the student presents a solution curve through the point $(0, 8)$ and a solution curve through the point $(3, 2)$. Both curves have the general form required relative to the slope field, and the student earned both points. In part (b) the student earned the first point for correctly demonstrating two steps of Euler's method using the initial value $f(0) = 8$ and a step size of $\Delta t = \frac{1}{2}$ to approximate $f(1)$. The student makes an error in writing 7 as $\frac{132}{16}$ and did not earn the second point. In part (c) 2 points were earned when the student correctly finds $\frac{d^2y}{dt^2}$. The student evaluates $f'(0)$ and $f''(0)$. The student earned the third point with use of the declared values of $f'(0)$ and $f''(0)$ along with $f(0) = 8$ to correctly construct a second-degree Taylor polynomial for *f* about $t = 0$. The student makes an arithmetic error in the calculation of $f''(0)$ and did not earn the fourth point. In part (d) the student presents an incorrect range for *f* and did not earn the point.

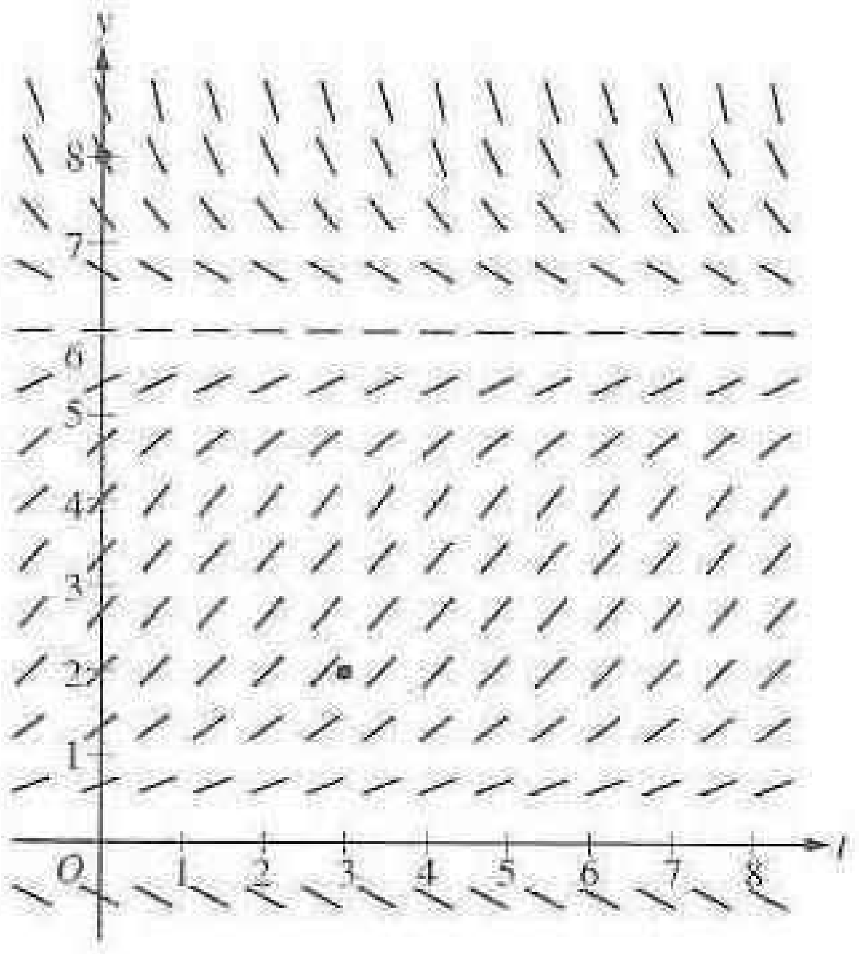
Student Response 3 (Score: 3)

The student earned 3 points: 2 points in part (a), 1 point in part (b), no points in part (c), and no points in part (d). In part (a) the student presents a solution curve through the point $(0, 8)$ and a solution curve through the point $(3, 2)$. Both curves have the general form required relative to the slope field, and the student earned both points. In part (b) the student earned the first point for correctly demonstrating two steps of Euler's method using the initial value $f(0) = 8$ and a step size of $\Delta t = \frac{1}{2}$ to approximate $f(1)$. The student makes a copying error in the second step of Euler's method by using $y = -7$ instead of $y = 7$, and the second point was not earned. In part (c) the expression for $\frac{d^2y}{dt^2}$ is not correct, and the student did not earn the first 2 points. The student provides a formula for the second-degree Taylor polynomial but does not calculate values of $f'(0)$ and $f''(0)$ to construct a Taylor polynomial that could be used to approximate $f(1)$. The student was not eligible for the third point and, therefore, was not eligible for the fourth point. In part (d) the student presents an incorrect range for *f* and did not earn the point.

Scoring Guidelines for Calculus BC Question 6

Consider the logistic differential equation $\frac{dy}{dt} = \frac{y}{8}(6 - y)$. Let $y = f(t)$ be the particular solution to the differential equation with $f(0) = 8$.

(a) A slope field for this differential equation is given below. Sketch possible solution curves through the points $(3, 2)$ and $(0, 8)$.
(Note: Use the axes provided in the exam booklet.)

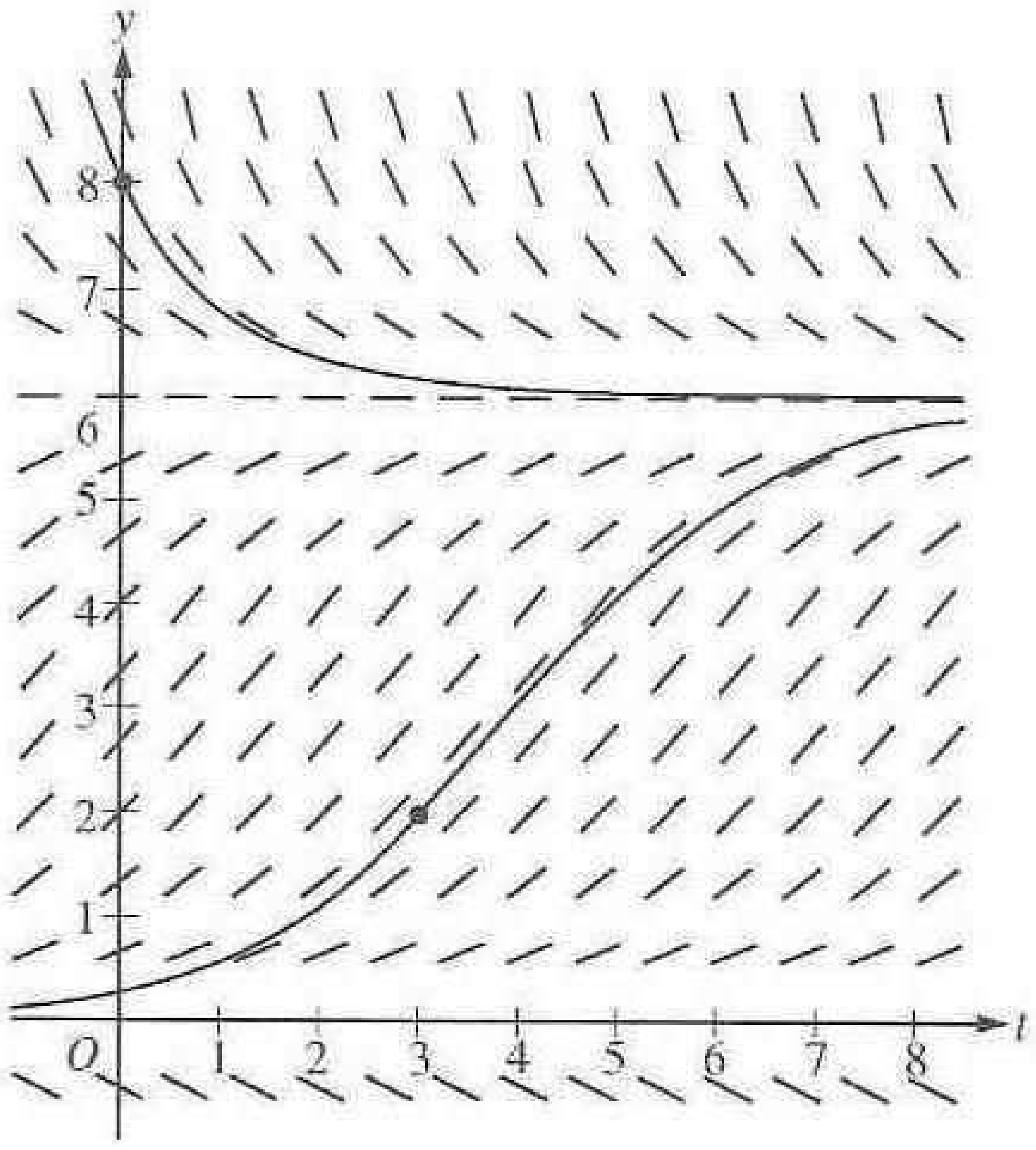


(b) Use Euler's method, starting at $t = 0$ with two steps of equal size, to approximate $f(1)$.

(c) Write the second-degree Taylor polynomial for f about $t = 0$, and use it to approximate $f(1)$.

(d) What is the range of f for $t \geq 0$?

(a)



$2 : \begin{cases} 1\text{: solution curve through } (0,8) \\ 1\text{: solution curve through } (3,2) \end{cases}$

(b) $f\left(\frac{1}{2}\right) \approx 8 + (-2)\left(\frac{1}{2}\right) = 7$

$f(1) \approx 7 + \left(-\frac{7}{8}\right)\left(\frac{1}{2}\right) = \frac{105}{16}$

$2 : \begin{cases} 1 : \text{Euler's method with two steps} \\ 1 : \text{approximation of } f(1) \end{cases}$

(c) $\frac{d^2y}{dt^2} = \frac{1}{8}\frac{dy}{dt}(6 - y) + \frac{y}{8}\left(-\frac{dy}{dt}\right)$

$f(0) = 8;\ f'(0) = \left.\frac{dy}{dt}\right|_{t=0} = \frac{8}{8}(6 - 8) = -2;$ and

$f''(0) = \left.\frac{d^2y}{dt^2}\right|_{t=0} = \frac{1}{8}(-2)(-2) + \frac{8}{8}(2) = \frac{5}{2}$

The second-degree Taylor polynomial for f about $t = 0$ is $P_2(t) = 8 - 2t + \frac{5}{4}t^2$.

$f(1) \approx P_2(1) = \frac{29}{4}$

$4 : \begin{cases} 2 : \frac{d^2y}{dt^2} \\ 1 : \text{second-degree Taylor polynomial} \\ 1 : \text{approximation of } f(1) \end{cases}$

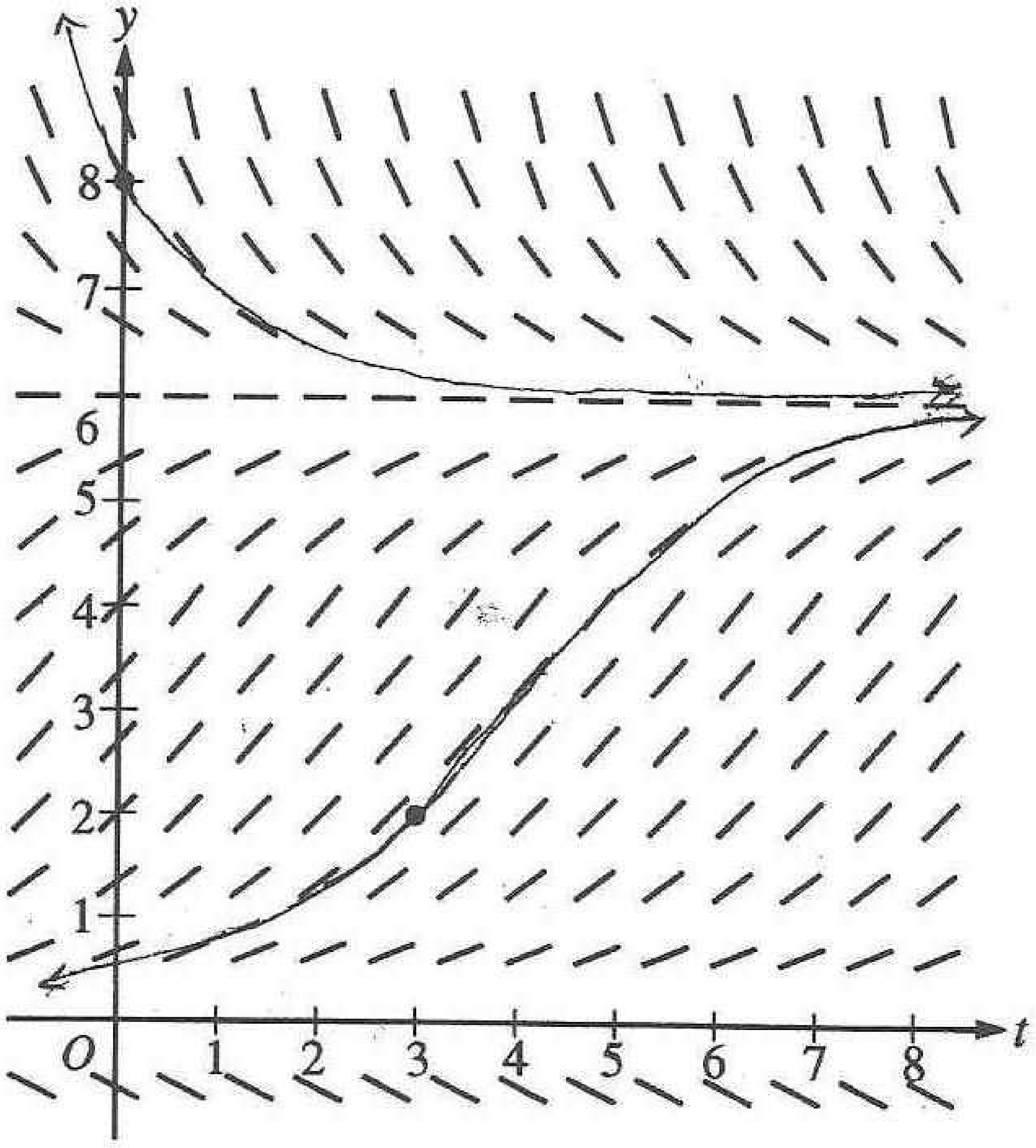
(d) The range of f for $t \geq 0$ is $6 < y \leq 8$.

1 : answer

Sample Student Responses for Calculus BC Question 6

Student Response 1 (Score: 9)

Work for problem 6(a)



Work for problem 6(b)

$y = f(t)$

t	0	0.5	1
$f(t)$	8	7	$\frac{105}{16}$
$f'(t)$	-2	$-\frac{7}{8}$	

$\frac{dy}{dt} \approx \frac{8}{8}(6-8) = 1(-2) = -2$

$(-2 \times 0.5) + 8 = -1 + 8 = 7$

$\frac{dy}{dt} = \frac{7}{8}(6-7) = \frac{7}{8}(-1) = -\frac{7}{8}$

$(-\frac{7}{8} \times \frac{1}{2}) + 7 = -\frac{7}{16} + \frac{112}{16}$

$= \frac{105}{16}$

$\boxed{f(1) \approx \frac{105}{16}}$

Student Response 1 *(continued)*

Work for problem 6(c)

$$\frac{dy}{dt} = f'(0) = \frac{8}{8}(6-8) = -2$$

$$\frac{d^2y}{dt^2} = \frac{1}{8}\frac{dy}{dt}(6-y) + \frac{y}{8}\left(-\frac{dy}{dt}\right)$$

$$f''(0) = \frac{1}{8}(-2)\cdot(6-8) + \frac{8}{8}(+(+2)) = -\frac{1}{4}(-2) + 1(2)$$

$$= +\frac{1}{2} + 2 = \frac{5}{2}$$

$$P(t) = f(0) + f'(0)(t-0) + \frac{f''(0)\,(t-0)^2}{2!}$$

$$= 8 + (-2)t + \frac{5}{2}\frac{t^2}{2!}$$

$$P(t) = 8 - 2t + \frac{5t^2}{4}$$

$$f(1) \simeq P(1) = 8 - 2 + \frac{5}{4} = 6 + \frac{5}{4} = \frac{24+5}{4} = \frac{29}{4}$$

$$\boxed{f(1) \simeq \frac{29}{4}}\ \boxed{\simeq 7.25}$$

Work for problem 6(d)

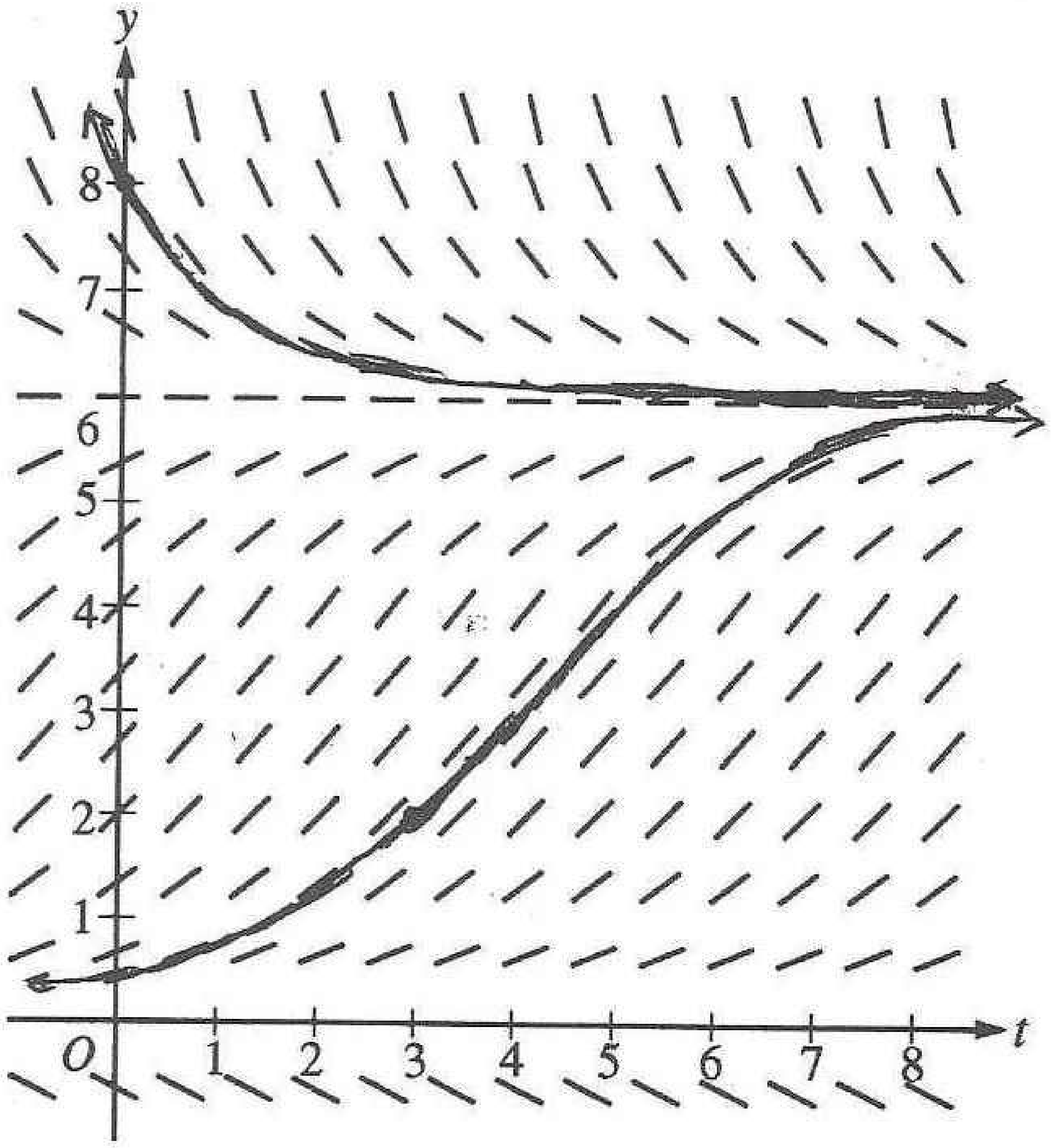
For this logistic equation, the carrying capacity is <u>6</u>.

$\therefore$ Range f : $6 < f(t) \leq 8$, for $t \geq 0$

(Note : $\lim_{t\to\infty} f(t) = 6$)

Student Response 2 (Score: 6)

Work for problem 6(a)



Work for problem 6(b)

$$\frac{dy}{dt} = \frac{y}{8}(6-y)$$

$$f(0) = 8$$

$$y_{.5} = 8 + .5\,f'(0,8)$$
$$= 8 + .5\,(-2)$$
$$= 8 - 1 \quad = 7$$

$$\begin{array}{r} {}^{4} \\ 16 \\ \times\ 7 \\ \hline 132 \end{array}$$

$$y_1 = 7 + .5\,f'(.5,7)$$
$$= 7 + .5\left(\frac{-7}{8}\right)$$
$$= 7 - \frac{7}{16}$$
$$= \frac{132}{16} - \frac{7}{16}$$
$$= \frac{125}{16}$$

Student Response 2 *(continued)*

Work for problem 6(c)

$$f(x) \approx 8 - 2x + \frac{3}{4}x^2$$

$$f(1) \approx 8 - 2 + \frac{3}{4}$$

$$\approx \frac{24}{4} + \frac{3}{4}$$

$$\approx \frac{27}{4}$$

$$\frac{d^2y}{dt^2} = \frac{y'}{8}(6-y) - \frac{y'y}{8}$$

$$= \frac{-2}{8}(6-8) + 2$$

$$= \frac{2}{4} + 2$$

$$= \frac{3}{2}$$

Work for problem 6(d)

$$f(x) : \overset{\text{range}}{[8, 6]} \rightarrow \lim_{t\to\infty} \frac{y}{8}(6-y) = 6$$

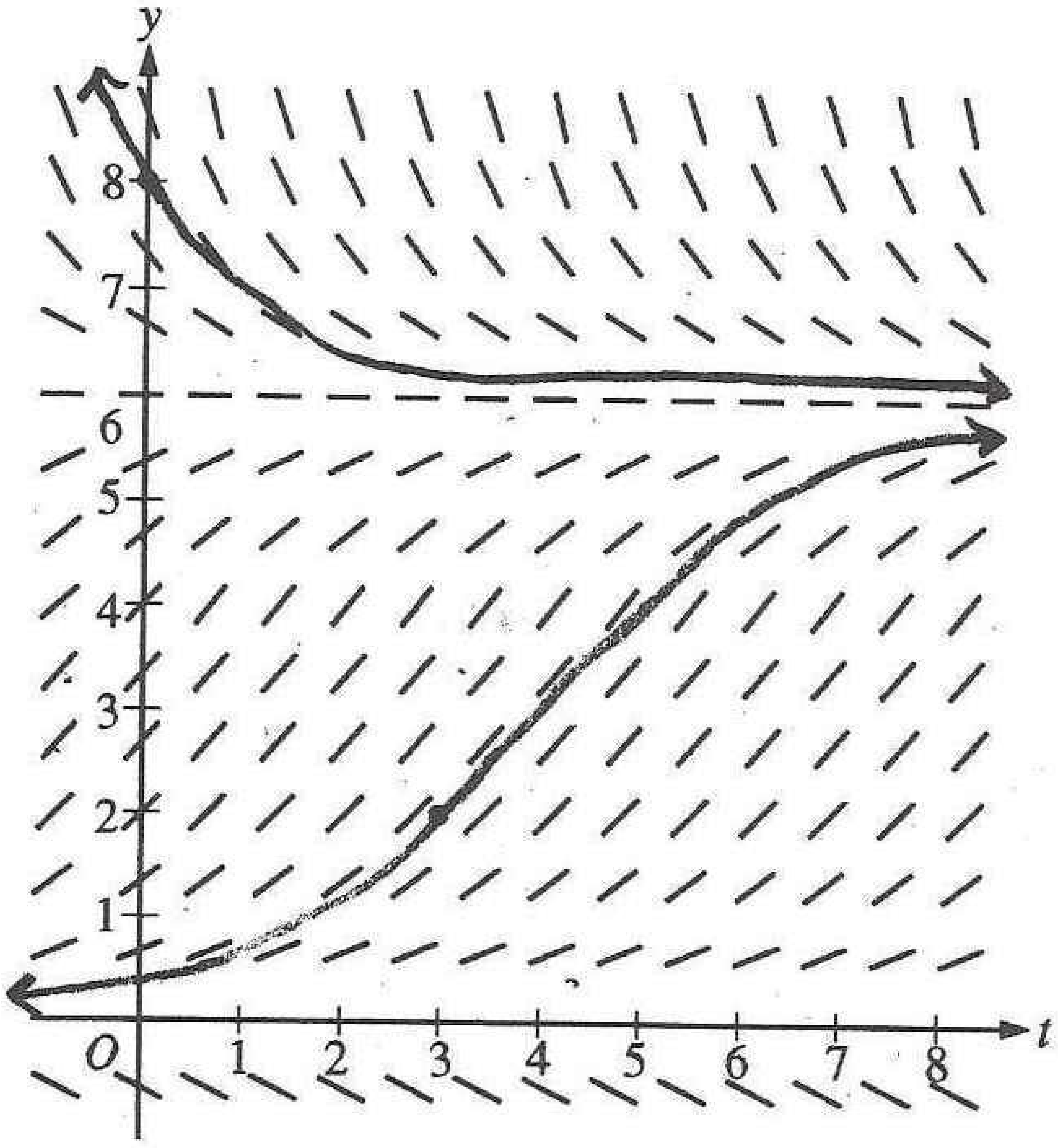
$\uparrow$ Limit

$\downarrow$

$f(6) = 8$

Student Response 3 (Score: 3)

Work for problem 6(a)



Work for problem 6(b)

Initial t	Initial y	Step Size	Final t	Final y
0	8	0.5	0.5	7
0.5	-7	0.5	1	

$y_1 \approx 8 + \left(\frac{8}{8}(6-8)\right)(0.5)$

$\approx 8 + -1 \approx 7$

$y_2 \approx 7 + \left(\frac{-7}{8}(6-7)\right)(0.5)$

$\approx 7 + \left(\frac{7}{8}\right)\left(\frac{1}{2}\right)$

$\approx 7 + \frac{7}{16}$

$f(1) \approx \frac{119}{16}$

Work for problem 6(c)

$$\sum_{0}^{\infty} \frac{f^n(0)\,x^n}{n!} = 8 + \left(\frac{y}{8}(6-y)\right)(t) + \frac{\left(\frac{3y^2}{8} - \frac{y^3}{24}\right)(t)^2}{2!}$$

$$= 8 + \left(\frac{y}{8}(6-y)\right)(1) + \frac{\left(\frac{3y^2}{8} - \frac{y^3}{24}\right)(1)^2}{2!}$$

$$= 8 + \frac{y}{8}(6-y) + \frac{\frac{3y^2}{8} - \frac{y^3}{24}}{2}$$

$$= 8 + \frac{y}{8}(6-y) + \frac{3y^2}{16} - \frac{y^3}{48}$$

$$= 8 + \frac{3y}{4} - \cancel{\frac{2y^2}{16}} + \frac{\overset{1}{\cancel{3}}y^2}{16} - \frac{y^3}{48}$$

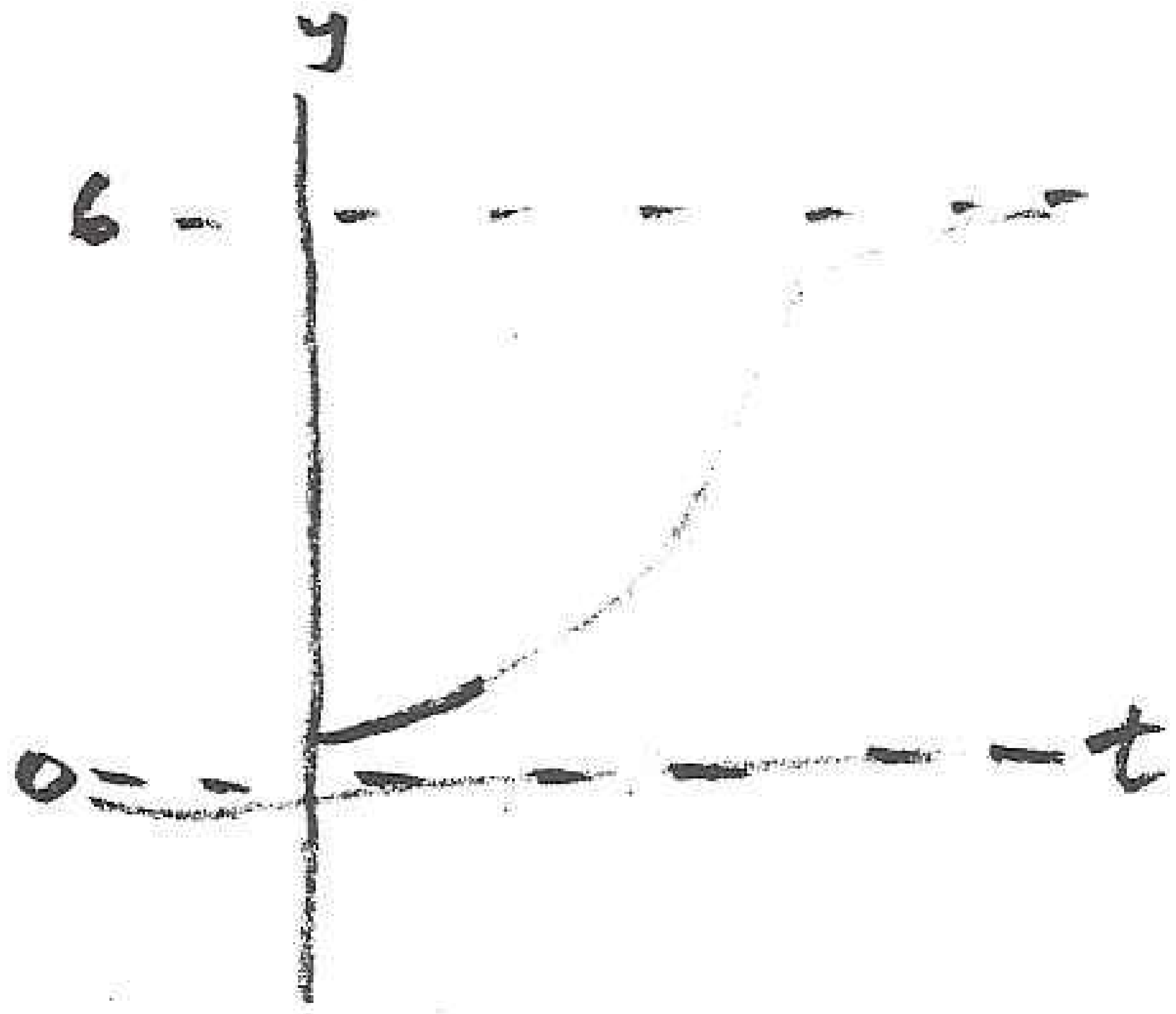
$$= 8 + \frac{3y}{4} + \frac{y^2}{16} - \frac{y^3}{48}$$

$$\frac{dy}{dt} = \frac{3y}{4} - \frac{y^2}{8} \qquad \frac{dy^2}{dt^2} = \frac{3y^2}{8} - \frac{y^3}{24}$$

Work for problem 6(d)

Range of f for $t \geq 0$

$(0, 6)$



Chapter IV: Statistical Information

This chapter presents statistical information about overall student performance on the 2008 AP Calculus AB and Calculus BC Exams.

Tables 4.1AB and 4.1BC show and summarize score distributions for each of the free-response questions. The scoring worksheets presented in Tables 4.2AB and 4.2BC provide step-by-step instruction for calculating AP section and composite scores and converting composite scores to AP grades. (Table 4.2AB1 is for the Calculus AB subscore grade.) Tables 4.3AB and 4.3BC include distributions for the overall exam grades. (Table 4.3AB1 is for the Calculus AB subscore grade.) The grade distributions conditioned on multiple-choice performance presented in Tables 4.4AB and 4.4BC are useful in estimating a student's AP grade given only the student's multiple-choice score.

College comparability studies, which are conducted to collect information for setting AP grade cut-points, are briefly discussed in this chapter. In addition, the purpose and intended use of AP Exams are reiterated to promote appropriate interpretation and use of the AP Exam and exam results.

Table 4.1AB—Calculus AB Section II Scores

The following table shows the score distributions for AP students on each free-response question from the 2008 AP Calculus AB Exam.

Score	Question 1		Question 2		Question 3	
	No. of Students	% at Score	No. of Students	% at Score	No. of Students	% at Score
9	21,008	9.8	1,351	0.6	715	0.3
8	13,273	6.2	9,878	4.6	8,764	4.1
7	35,732	16.6	15,513	7.2	9,206	4.3
6	23,108	10.7	26,366	12.3	9,569	4.5
5	32,269	15.0	25,294	11.8	15,531	7.2
4	24,748	11.5	26,898	12.5	22,795	10.6
3	22,432	10.4	20,698	9.6	27,376	12.7
2	13,770	6.4	24,970	11.6	17,876	8.3
1	12,457	5.8	17,570	8.2	42,721	19.9
0	12,142	5.7	37,613	17.5	46,465	21.6
No Response	4,125	1.9	8,913	4.1	14,046	6.5
Total Students	215,064		215,064		215,064	
Mean	4.89		3.36		2.45	
Standard Deviation	2.61		2.55		2.39	
Mean as % of Maximum Score	54		37		27	

Score	Question 4		Question 5		Question 6	
	No. of Students	% at Score	No. of Students	% at Score	No. of Students	% at Score
9	5,043	2.3	3,492	1.6	4,578	2.1
8	7,041	3.3	15,057	7.0	12,556	5.8
7	10,840	5.0	26,921	12.5	15,225	7.1
6	13,938	6.5	27,508	12.8	16,438	7.6
5	14,511	6.8	19,722	9.2	19,552	9.1
4	18,341	8.5	13,758	6.4	21,601	10.0
3	21,710	10.1	10,045	4.7	19,597	9.1
2	27,003	12.6	40,217	18.7	25,721	12.0
1	32,787	15.3	25,772	12.0	17,625	8.2
0	53,882	25.1	24,306	11.3	47,104	21.9
No Response	9,968	4.6	8,266	3.8	15,067	7.0
Total Students	215,064		215,064		215,064	
Mean	2.60		3.70		3.06	
Standard Deviation	2.58		2.73		2.75	
Mean as % of Maximum Score	29		41		34	

Table 4.2AB—Calculus AB Scoring Worksheet

Section I: Multiple Choice

[______________ − (1/4 × ______________)] × 1.2272 = ______________

Number Correct (out of 44)

Number Wrong

Weighted Section I Score (If less than zero, enter zero; do not round)

Section II: Free Response

Question 1 ______________ (out of 9) × 1.0000 = ______________ (Do not round)

Question 2 ______________ (out of 9) × 1.0000 = ______________ (Do not round)

Question 3 ______________ (out of 9) × 1.0000 = ______________ (Do not round)

Question 4 ______________ (out of 9) × 1.0000 = ______________ (Do not round)

Question 5 ______________ (out of 9) × 1.0000 = ______________ (Do not round)

Question 6 ______________ (out of 9) × 1.0000 = ______________ (Do not round)

Sum = ______________

Weighted Section II Score (Do not round)

AP Grade Conversion Chart Calculus AB

Composite Score Range	AP Grade
65–108	5
48–64	4
34–47	3
21–33	2
0–20	1

Composite Score

______________ + ______________ = ______________

Weighted Section I Score

Weighted Section II Score

Composite Score (Round to nearest whole number)

Table 4.3AB—Calculus AB Grade Distributions

More than 60 percent of the AP students who took this exam earned a qualifying grade of 3 or above.

	Exam Grade	Number of Students	Percent at Grade
Extremely well qualified	5	46,963	21.8
Well qualified	4	45,676	21.2
Qualified	3	38,490	17.9
Possibly qualified	2	32,777	15.2
No recommendation	1	51,158	23.8
Total Number of Students		215,064	
Mean Grade		3.02	
Standard Deviation		1.48	

Table 4.4AB—Calculus AB Section I Scores and AP Grades

For a given range of multiple-choice scores, this table shows the percentage of students receiving each AP grade. If you hav calculated the multiple-choice score (**Weighted Section I Score**) by using the formula shown in Table 4.2AB, you can use this table to figure out the most likely grade that the student would receive based only on that multiple-choice score.

	AP Grade					
Multiple-Choice Score	**1**	**2**	**3**	**4**	**5**	**Total**
46 to 54	0.0%	0.0%	0.0%	0.1%	99.9%	3.8%
37 to 45	0.0%	0.0%	0.1%	8.0%	91.9%	13.5%
28 to 36	0.0%	0.1%	7.0%	65.6%	27.3%	20.4%
19 to 27	0.0%	10.5%	58.4%	30.7%	0.3%	21.6%
10 to 18	20.0%	59.9%	19.5%	0.6%	0.0%	19.4%
0 to 9	93.5%	6.3%	0.2%	0.0%	0.0%	21.3%
Total	23.8%	15.2%	17.9%	21.2%	21.8%	100.0%

Table 4.1BC—Calculus BC Section II Scores

The following table shows the score distributions for AP students on each free-response question from the 2008 AP Calculus BC Exam.

Score	Question 1		Question 2		Question 3		Question 4	
	No. of Students	% at Score	No. of Students	% at Score	No. of Students	% at Score	No. of Students	% at Score
9	15,762	24.1	1,150	1.8	319	0.5	4,510	6.9
8	8,275	12.7	8,682	13.3	2,770	4.2	4,820	7.4
7	12,487	19.1	10,387	15.9	11,612	17.8	7,034	10.8
6	7,566	11.6	12,966	19.8	14,862	22.7	7,527	11.5
5	7,497	11.5	9,516	14.6	10,469	16.0	6,749	10.3
4	5,337	8.2	7,565	11.6	5,187	7.9	7,293	11.2
3	3,635	5.6	4,819	7.4	3,465	5.3	7,452	11.4
2	2,453	3.8	4,180	6.4	3,345	5.1	7,104	10.9
1	1,553	2.4	2,328	3.6	3,270	5.0	5,762	8.8
0	642	1.0	3,117	4.8	6,814	10.4	6,262	9.6
No Response	180	0.3	677	1.0	3,274	5.0	874	1.3
Total Students	65,387		65,387		65,387		65,387	
Mean	6.38		5.10		4.42		4.26	
Standard Deviation	2.30		2.30		2.54		2.73	
Mean as % of Maximum Score	71		57		49		47	

Score	Question 5 (AB Part)		Question 5 (BC Part)		Question 6	
	No. of Students	% at Score	No. of Students	% at Score	No. of Students	% at Score
9					1,218	1.9
8					2,999	4.6
7					2,291	3.5
6					3,321	5.1
5	13,046	20.0			10,136	15.5
4	21,762	33.3	27,540	42.1	18,783	28.7
3	14,863	22.7	11,650	17.8	7,569	11.6
2	6,027	9.2	8,263	12.6	14,955	22.9
1	5,709	8.7	4,163	6.4	1,738	2.7
0	3,171	4.9	11,032	16.9	1,539	2.4
No Response	809	1.2	2,739	4.2	838	1.3
Total Students	65,387		65,387		65,387	
Mean	3.28		2.54		3.84	
Standard Deviation	1.43		1.58		1.92	
Mean as % of Maximum Score	66		64		43	

4.2BC—Calculus BC Scoring Worksheet

Section I: Multiple Choice

[__________ − (1/4 × __________)] × 1.2000 = __________

Number Correct (out of 45)

Number Wrong

Weighted Section I Score (If less than zero, enter zero; do not round)

Section II: Free Response

Question 1 __________ × 1.0000 = __________
(out of 9) (Do not round)

Question 2 __________ × 1.0000 = __________
(out of 9) (Do not round)

Question 3 __________ × 1.0000 = __________
(out of 9) (Do not round)

Question 4 __________ × 1.0000 = __________
(out of 9) (Do not round)

Question 5 (AB Part) __________ × 1.0000 = __________
(out of 5) (Do not round)

Question 5 (BC Part) __________ × 1.0000 = __________
(out of 4) (Do not round)

Question 6 __________ × 1.0000 = __________
(out of 9) (Do not round)

Sum = __________
Weighted Section II Score (Do not round)

AP Grade Conversion Chart
Calculus BC

Composite Score Range	AP Grade
66–108	5
55–65	4
39–54	3
31–38	2
0–30	1

Composite Score

__________ + __________ = __________

Weighted Section I Score

Weighted Section II Score

Composite Score (Round to nearest whole number)

Table 4.3BC—Calculus BC Grade Distributions

Nearly 80 percent of the AP students who took this exam earned a qualifying grade of 3 or above.

	Exam Grade	Number of Students	Percent at Grade
Extremely well qualified	5	27,964	42.8
Well qualified	4	11,469	17.5
Qualified	3	12,829	19.6
Possibly qualified	2	4,484	6.9
No recommendation	1	8,641	13.2
Total Number of Students		65,387	
Mean Grade		3.70	
Standard Deviation		1.41	

Table 4.4BC—Calculus BC Section I Scores and AP Grades

For a given range of multiple-choice scores, this table shows the percentage of students receiving each AP grade. If you have calculated the multiple-choice score (**Weighted Section I Score**) by using the formula shown in Table 4.2BC, you can use this table to figure out the most likely grade that the student would receive based only on that multiple-choice score.

Multiple-Choice Score	AP Grade					Total
	1	2	3	4	5	
46 to 54	0.0%	0.0%	0.0%	0.0%	100.0%	9.4%
37 to 45	0.0%	0.0%	0.1%	2.4%	97.5%	21.6%
28 to 36	0.0%	0.1%	7.6%	44.3%	48.1%	25.1%
19 to 27	0.5%	5.6%	64.6%	28.0%	1.3%	20.9%
10 to 18	33.0%	37.4%	29.2%	0.4%	0.0%	14.1%
0 to 9	94.3%	4.5%	1.1%	0.1%	0.0%	9.0%
Total	13.2%	6.9%	19.6%	17.5%	42.8%	100.0%

Table 4.2AB1—Calculus AB Subscore Scoring Worksheet

Section I: Multiple Choice

Questions (2, 6, 8, 9, 10, 13, 14, 15, 17, 18, 21, 25, 27, 76, 77, 78, 80, 81, 83, 85–92)

[__________ − (1/4 × __________)] × 1.1851 = __________

Number Correct (out of 27)

Number Wrong

Weighted Section I Score (If less than zero, enter zero; do not round)

Section II: Free Response

Question 1 __________ (out of 9) × 1.0000 = __________ (Do not round)

Question 2 __________ (out of 9) × 1.0000 = __________ (Do not round)

Question 4 __________ (out of 9) × 1.0000 = __________ (Do not round)

Question 5 (AB Part) __________ (out of 5) × 1.0000 = __________ (Do not round)

Sum = __________ Weighted Section II Score (Do not round)

AP Grade Conversion Chart
Calculus AB Subscore

Composite Score Range	AP Grade
40–64	5
29–39	4
20–28	3
12–19	2
0–11	1

Composite Score

__________ + __________ = __________

Weighted Section I Score

Weighted Section II Score

Composite Score (Round to nearest whole number)

Table 4.3AB1—Calculus AB Subscore Grade Distributions

More than 86 percent of the AP students who took this exam earned an AB subscore grade of 3 or above.

	Exam Grade	Number of Students	Percent at Grade
Extremely well qualified	5	32,440	49.6
Well qualified	4	15,561	23.8
Qualified	3	8,466	13.0
Possibly qualified	2	4,992	7.6
No recommendation	1	3,928	6.0
Total Number of Students		65,387	
Mean Grade		4.03	
Standard Deviation		1.21	

How AP Grades Are Determined

As described in Chapter II, the AP Calculus Exams consist of two sections. Section I originally consisted of 45 multiple-choice questions for both exams, but for the Calculus AB Exam, item 19 was not scored. Therefore, the scores for Calculus AB range from a minimum possible score of 0 to a maximum score of 44 points, while the Calculus BC Exam has scores that range from a minimum possible score of 0 to a maximum possible score of 45 points. Section II, which consists of 6 free-response questions, has scores that range from a minimum possible score of 0 to a maximum possible score of 9 points for each question.

The scores on the different parts of the exam are combined to produce a composite score for each student that ranges from a minimum possible score of 0 to a maximum possible score of 108 points. In calculating the composite scores, scores on the different parts are multiplied by weights.

Composite scores are not released to the student, school, or college. Instead, the composite scores are converted to grades on an AP 5-point scale, and it is these grades that are reported. The process of calculating the composite score and converting it to a grade involves a number of steps, which are shown in the Scoring Worksheets (Tables 4.2AB, 4.2BC, and 4.2AB1) and described in detail here.

1. **The score on Section I is calculated.** In calculating the score for Section I, a fraction of the number of wrong answers is subtracted from the number of right answers. With this adjustment to the number of right answers, students are not likely to benefit from random guessing. The value of the fraction is 1/4 for the five-choice questions in the AP Calculus Exams. The maximum possible weighted score on Section I is 54 points, and it accounts for 50 percent of the maximum possible composite score.
2. **The score on Section II is calculated.** The 6 questions in Section II are weighted equally. The weighted scores on the questions of Section II are summed to give the total weighted score for Section II. The maximum possible weighted score on Section II is 54 points, and it accounts for 50 percent of the maximum possible composite score.
3. **AP grades are calculated.** Composite scores are calculated by adding the weighted Section I and weighted Section II scores together. The AP grades are calculated by comparing the composite scores to the four composite cut-scores selected during the grade-setting process. A variety of information is available during the grade-setting process to help determine the cut-scores corresponding to each AP grade:
   - Statistical information based on test score equating
   - College/AP grade comparability studies, if available
   - The Chief Reader's observations of students' free-response performance
   - The distribution of scores on different parts of the exam
   - AP grade distributions from the past three years

See Tables 4.3AB, 4.3BC, and 4.3AB1 for the grade distributions for the 2008 AP Calculus AB and BC Exams.

If you are interested in more detailed information about this process, please visit AP Central (apcentral.collegeboard.com). There you will also find information about how the AP Exams are developed, how validity and reliability studies are conducted, and other data on all AP subjects.

College Comparability Studies

The Advanced Placement Program has conducted college grade comparability studies in all AP subjects. These studies have compared the performance of AP students with that of college students in related courses who have taken the AP Exam at the end of their course. In general, AP cut-points are selected so that the lowest AP 5 is equivalent to the average A in college, the lowest AP 4 is equivalent to the average B, and the lowest AP 3 is equivalent to the average C (see below).

AP Grade	Average College Grade
5	
	A
4	
	B
3	
	C
2	
	D
1	

Research studies conducted by colleges and universities and by the AP Program indicate that AP students generally receive higher grades in advanced courses than do students who have taken the regular first-year courses at the institution. Colleges and universities are encouraged to periodically undertake such studies to establish appropriate policy for accepting AP grades and ensure that admissions and placement standards remain valid. It is critical to verify that admissions and placement measures established for a previous class continue for future classes. Summaries of several studies are available at AP Central. Also on the College Board Web site is the free Admitted Class

Evaluation Service™ (http://professionals.collegeboard.com/higher-ed/validity) that can predict how admitted college students will perform at a particular institution generally and how successful they can be in specific classes.

Reminders for All Grade Report Recipients

AP Exams are designed to provide accurate assessments of achievement. However, any exam has limitations, especially when used for purposes other than those intended. Presented here are some suggestions for teachers to aid in the use and interpretation of AP grades:

- AP Exams in different subjects are developed and evaluated independently of each other. They are linked only by common purpose, format, and method of reporting results. Therefore, comparisons should not be made between grades on different AP Exams. An AP grade in one subject may not have the same meaning as the same AP grade in another subject, just as national and college standards vary from one discipline to another.
- Grade reports are confidential. Everyone who has access to AP grades should be aware of the confidential nature of the grades and agree to maintain their security. In addition, school districts and states should not release data about high school performance without the school's permission.
- AP Exams are not designed as instruments for teacher or school evaluation. Many factors influence AP Exam performance in a particular course or school in any given year. Thus, differences in AP Exam performance should be carefully studied before being attributed to the teacher or school.
- Where evaluation of AP students, teachers, or courses is desired, local evaluation models should be developed. An important aspect of any evaluation model is the use of an appropriate method of comparison or frame of reference to account for yearly changes in student composition and ability, as well as local differences in resources, educational methods, and socioeconomic factors.
- The *AP Instructional Planning Report* is sent to schools automatically and can be a useful diagnostic tool in reviewing course results. This report identifies areas of strength and weakness for the students in each AP course. The information may also provide teachers with guidance for course emphasis and student evaluation.
- Many factors can influence exam results. AP Exam performance can be affected by the degree of agreement between a course and the course defined in the relevant AP Course Description, use of different instructional methods, differences in emphasis or preparation on particular parts of the exam, differences in curriculum, or differences in student background and preparation in comparison with the national group.

Reporting AP Grades

The results of AP Exams are disseminated in several ways to students, their secondary schools, and the colleges they select:

- College and student grade reports contain a cumulative record of all grades earned by the student on AP Exams during the current or previous years. These reports are sent in July. (School grade reports are sent shortly thereafter.)
- Group results for AP Exams are available to AP teachers in the *AP Instructional Planning Report* mentioned previously. This report provides useful information comparing local student performance with that of the total group of students taking an exam, as well as details on different subsections of the exam.

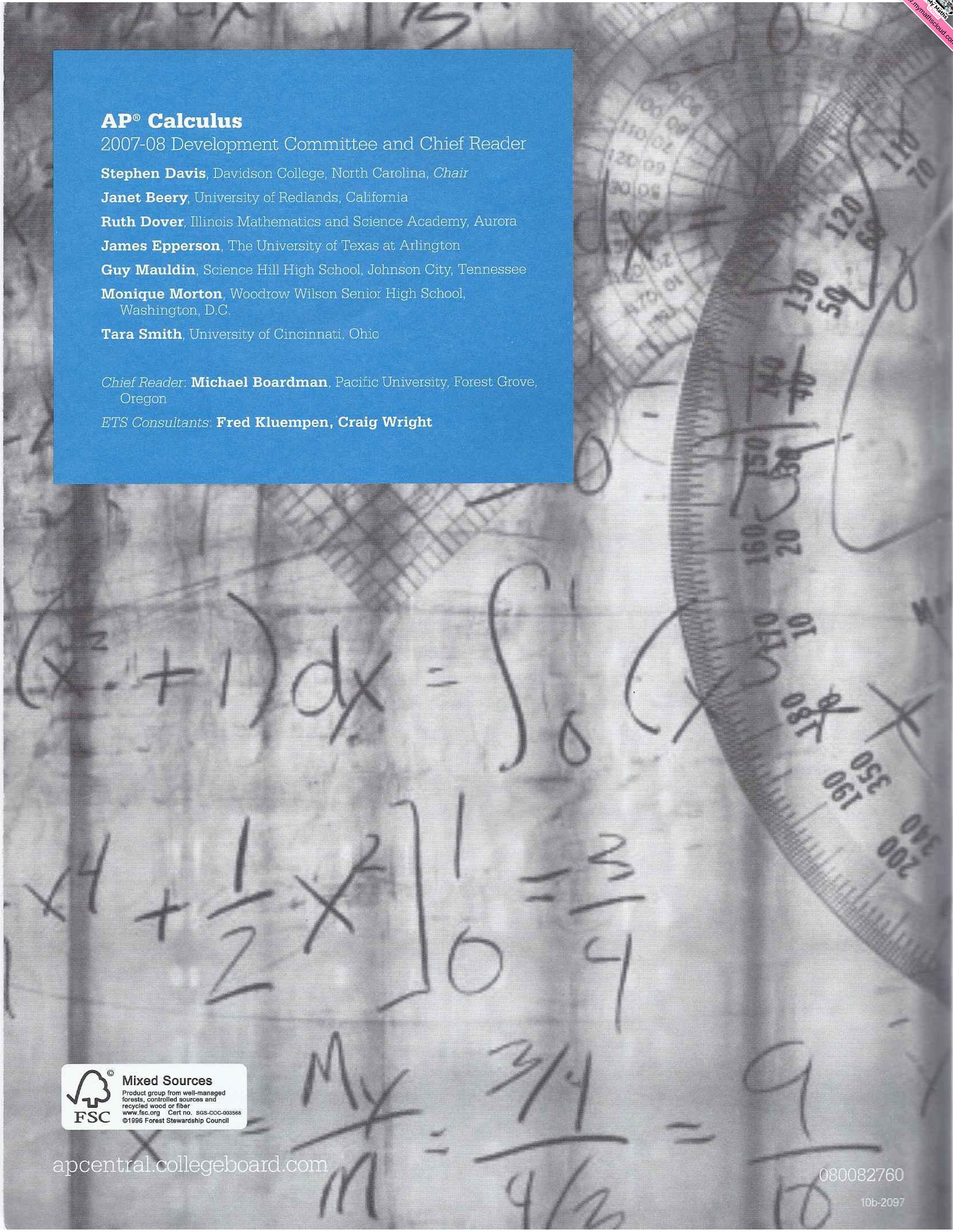
Several other reports produced by the AP Program provide summary information on AP Exams:

- State, National, and Canadian Reports show the distribution of grades obtained on each AP Exam for all students and for subsets of students broken down by gender and by ethnic group.
- The Program also produces a one-page summary of AP grade distributions for all exams in a given year.

For information on any of the above, please call AP Services at 609 771-7300 or e-mail apexams@info.collegeboard.org.

Purpose of AP Grades

AP grades are intended to allow participating colleges and universities to award college credit, advanced placement, or both to qualified students. In general, an AP grade of 3 or higher indicates sufficient mastery of course content to allow placement in the succeeding college course, or credit for and exemption from a college course comparable to the AP course. Students seeking credit through their AP grades should note that each college, not the AP Program or the College Board, determines the nature and extent of its policies for awarding advanced placement, credit, or both. Because policies regarding AP grades vary, students should consult the AP policy of individual colleges and universities. Students can find information in a college's catalog or Web site, or by using the AP Credit Policy search at www.collegeboard.com/ap/creditpolicy.



AP® Calculus

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